Islamic University of Gaza
Faculty of Engineering
Civil Engineering Department

SOLVED PROBLEMS IN FOUNDATION ENGINEERING

BY:

Prof. MOHAMMED A. AWAD

(Dean, Faculty of Engineering, Islamic University)

Third Edition
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مقدمة الطبعة الثانية

بسم الله الرحمن الرحيم

تقدم العلم في ميكانيكا التربة يدعونا إلى أن نتتبع باستمرار الجديد في عالم الأبحاث العلمية من أجل أن نلم بكل ما هو جديد في تطوير الأساسات ...

لهذا كانت الحاجة لمثل هذا العمل المتواضع الذي يعطي إشارة واضحة حول كيفية تطبيق الجديد في مفهوم سلوك التربة، آملين منكم أن تستفيدوا منها في حياتكم الأكاديمية والعملية.

ولا يفوتني هنا أن أنوه إلى مشاركة عدد من الأساتذة الأفاضل من جامعة الشرق الأوسط بتركيا وعلى رأسهم أ.عصمت أوردمير في إعداد هذا العمل.

وأتقدم بالشكر لكل من ساهم في الإعداد لهذا وخاصة م.أحمد أبو فول و م.فاتن بعلوشة.

مقدمة الطبعة الثالثة

استمرارا لعملية التطوير فلقد تم إضافة بعض المسائل في هذه الطبعة بمساعدة كل من المهندس لؤي القريناوي والمهندسة رنا جابر لذا أتقدم بالشكر الجزيل لمساهمتهم في إثراء هذا العمل ولكل من ساهم في الإعداد والطباعة.

TO THE SPIRIT OF:

PROF. ESSMAT ORDAMEIR

PROF. MOHAMMED AWAD

PREFACE

It is our great pleasure to introduce the third edition of Solved Problems in Foundation Engineering to our sincere students and engineers.

This book was prepared to cover the urgent need of the student of Civil Engineering Department at the Islamic University of Gaza to a reference gathering solutions to problems that may arise in the field of work and to help them to pass the course of Foundation Engineering (ECIV 4352) successfully.

This book contains different foundation engineering, problems such as:
Settlement under foundations, soil exploration, loads on foundations, different types of foundations, retaining walls, sheet piles and pile foundations.

A simple language was used in the preparation of this book so that our students can best understand foundation engineering problems.

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Problem # (1)

The subsoil profile of a very large area consists of a 6.0-m thick dense coarse sand layer overlaying a 9.0-m thick stratum of normally consolidated clay which in turn rests on a bed rock.

At a distance of 3.0-m and 6.0-m above the bed rock, the clay stratum contains thin horizontal layers of sand. It is expected that the sand layers provide free drainage for the clay above and below. The water table is 1.0-m below the ground surface. The coarse sand has a specific gravity (G_s) of 2.68 and porosity (n) of 0.3. The physical properties of clay layers are given below:

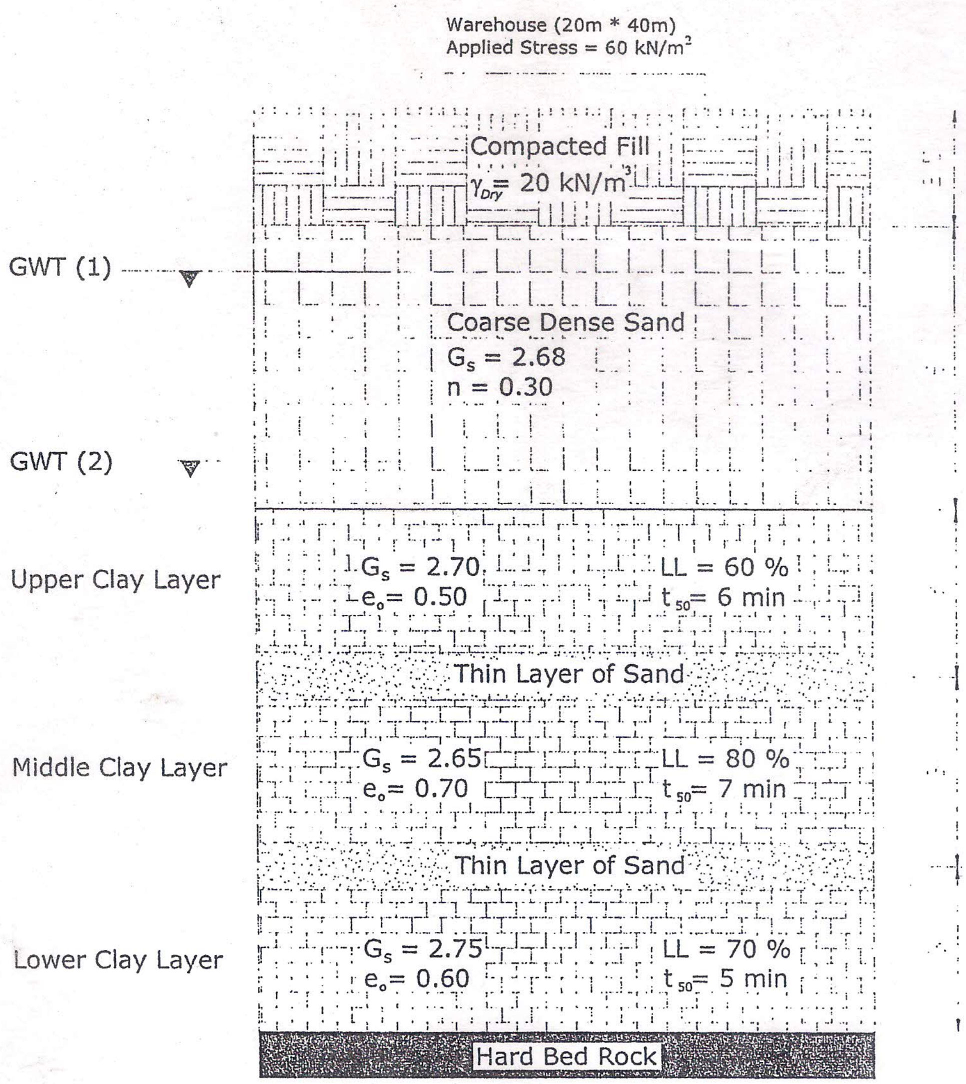
Layer	G_s	e_o	LL (%)	t_{50} (2.54 cm thick specimen)
Upper	2.70	0.50	60	6 min
Middle	2.65	0.70	80	7 min
Bottom	2.75	0.60	70	5 min

Over this area the following constructional operations are proposed and planned:

1. The ground water table is to be lowered to a depth of 5-m below the existing ground surface (Unit weight of water = 9.80-kN/m^3), coarse sand above water table is assumed to be dry.
2. A compacted granular fill is to be constructed over this large area with a unit weight of 20-kN/m^3 and a height of 2.5-m.
3. A warehouse of 20 * 40 m^2 is to be erected on the top of the new fill. A uniform load of 60-kN/m^2 is to be applied due to the construction of the warehouse.
4. The operation has been completed in 2-months period, which is very short, so that the loading can be considered instantaneous. Settlement of dense sand is not considered.

For the information given above, estimate:

A. The total settlement of the warehouse at both the center and the corners due to the consolidation of the clay layer
B. The time required for 80 % settlement under the center point of the warehouse



Useful formulas:

$$S = \left(\frac{C_c \times H}{1 + e_o}\right) \log \frac{P_o + \Delta P}{P_o}$$

For normally consolidated clay:

$$C_c = 0.009\,(LL - 10)$$

$$C_v = \frac{T_v \times H_{dr}^{\ 2}}{t}$$

$$\gamma_d = (1 - n) \times G_s \times \gamma_w$$

$$\gamma_{sat} = \frac{G_s \times \gamma_w + e \times \gamma_w}{1 + e}$$

$$\gamma_{sub} = \frac{(G_s - 1) \times \gamma_w}{1 + e}$$

$$e = \frac{n}{1 - n}$$

Wh re;

P_o is the present overburden pressure

ΔP added vertical pressure that causes settlement

LL is the liquid limit

C_c is the compression index

C_v is the coefficient of consolidation

T_v is the time factor

Solution:

1. **UNIT WEIGHTS OF SAND:**

$\gamma_{dry} = (1 - 0.3) * 2.68 * 9.80 = 18.4$ kPa

$\gamma_{sub} = 18.4 + (0.3 * 9.81) - 9.80 = 11.54$ kPa

2. **UNIT WEIGHTS OF CLAY LAYERS:**

$$Upper\ layer : \gamma_{sub} = \frac{2.70 - 1}{1 + 0.5} \times 9.80 = 11.1\, kPa$$

$$\text{Middle}\ layer : \gamma_{sub} = \frac{2.65 - 1}{1 + 0.7} \times 9.80 = 9.5\, kPa$$

$$Upper\ layer : \gamma_{sub} = \frac{2.75 - 1}{1 + 0.6} \times 9.80 = 10.7\, kPa$$

3. **ORIGINAL OVERBURDEN PRESSURE:**

At the center of the:		P_o (kPa)
Upper clay layer	$P_o = 1*18.4 + 5*11.5 + 1.5*11.1$	= 92.60
Middle clay layer	$P_o = 92.6 + 1.5*11.1 + 1.5*9.5$	= 123.6
Lower clay layer	$P_o = 123.6 + 1.5*9.5 + 1.5*10.7$	= 154.0

4. **COMPRESSION INDEX:**

Clay Layer	Cc = 0.009 (LL – 10)
The upper	$C_c = 0.009\ (60 - 10) = 0.45$
The middle	$C_c = 0.009\ (80 - 10) = 0.63$
The lower	$C_c = 0.009\ (70 - 10) = 0.54$

5. ADDED VERTICAL EFFECTIVE STRESS ΔP:

- Vertical stress increase due to the 4-m lowering of the water table:

$\Delta P = (\gamma_{dry} - \gamma_{sub}) * 4 = (18.4 - 11.5) * 4.0 = 27.6$ kPa

- Vertical stress increase due to the compacted fill:

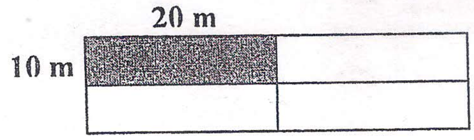
$\Delta P = 2.5 * 20 = 50$ kPa

- Vertical stress increase due to the construction of the warehouse:

$\Delta P = q * I_3$

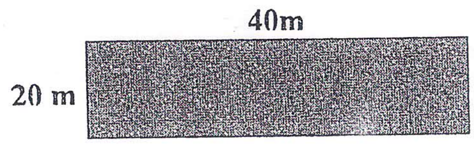
To estimate the vertical stress increase due to the construction of the warehouse, we have to do the following:

A. Beneath the center



Layer	Z (m)	m (B/Z)	n (L/Z)	I_3	$\Delta P = 4 * q * I_3$ (kPa)
Upper	10	1	2	0.200	4 * 60 * 0.200 = 48.0
Middle	13	0.77	1.54	0.173	4 * 60 * 0.173 = 41.5
Lower	16	0.63	1.25	0.148	4 * 60 * 0.148 = 35.5

B. Beneath the corners



Layer	Z (m)	m (B/Z)	n (L/Z)	I_3	$\Delta P = q * I_3$ (kPa)
Upper	10	2	4	0.239	60 * 0.239 = 14.3
Middle	13	1.54	3.08	0.230	60 * 0.230 = 13.8
Lower	16	1.25	2.5	0.217	60 * 0.217 = 13.0

6. SETTLEMENT:

6.1 SETTLEMENT BENEATH THE CENTER

Layer	Po	ΔP (kPa)	Settlement beneath the center (m)
Upper	92.60	27.6 + 50.0 + 48.0 = 125.6	$S=\frac{0.45\times 3.0}{1+0.5}\log\left(\frac{92.6+125.6}{92.6}\right)=0.34$
Middle	123.6	27.6 + 50.0 + 41.5 = 119.1	$S=\frac{0.63\times 3.0}{1+0.7}\log\left(\frac{123.6+119.1}{123.6}\right)=0.33$
Lower	154.0	27.6 + 50.0 + 35.5 = 113.1	$S=\frac{0.54\times 3.0}{1+0.6}\log\left(\frac{154+113.1}{154}\right)=0.24$

6.2 SETTLEMENT BENEATH THE CORNERS

Layer	Po	ΔP (kPa)	Settlement beneath the corners (m)
Upper	92.60	27.6 + 50.0 + 14.3 = 91.1	$S=\frac{0.45\times 3.0}{1+0.5}\log\left(\frac{92.6+91.1}{92.6}\right)=0.27$
Middle	123.6	27.6 + 50.0 + 13.8 = 91.4	$S=\frac{0.63\times 3.0}{1+0.7}\log\left(\frac{123.6+91.4}{123.6}\right)=0.27$
Lower	154.0	27.6 + 50.0 + 13.0 = 90.6	$S=\frac{0.54\times 3.0}{1+0.6}\log\left(\frac{154+90.6}{154}\right)=0.20$

7. TOTAL SETTLEMENT:

	Upper Layer	Middle Layer	Lower Layer	Total (m)
Center	0.34 (m)	0.33 (m)	0.24 (m)	**0.91**
Corner	0.27 (m)	0.27 (m)	0.20 (m)	**0.74**

8. DIFFERENTIAL SETTLEMENT:

$\Delta S = 0.91 - 0.74 = 0.17$ m

9. COEFFICIENT OF CONSOLIDATION:

Layer	H_{dr} (cm)	t_{50} (min)	T_v	$C_v=\frac{T_v\times H_{dr}^2}{t}\ cm^2/\min$
Upper	2.54 / 2 = 1.27	6.0	0.197	**0.0530**
Middle	2.54 / 2 = 1.27	7.0	0.197	**0.0454**
Lower	2.54 / 2 = 1.27	5.0	0.197	**0.0635**

10. TIME FOR 80% SETTLEMENT:

Layer	H_{dr} (cm)	T_v	C_v (cm^2/min)	$t = \frac{T_v \times H_{dr}^2}{C_v \times 60 \times 24 \times 30}$ month	Time (month)
Upper	$300/2 = 150$	0.565	0.0530	9.82 T_v	5.55
Middle	$300/2 = 150$	0.565	0.0454	11.47 T_v	6.48
Lower	300	0.565	0.0635	32.81 T_v	18.5

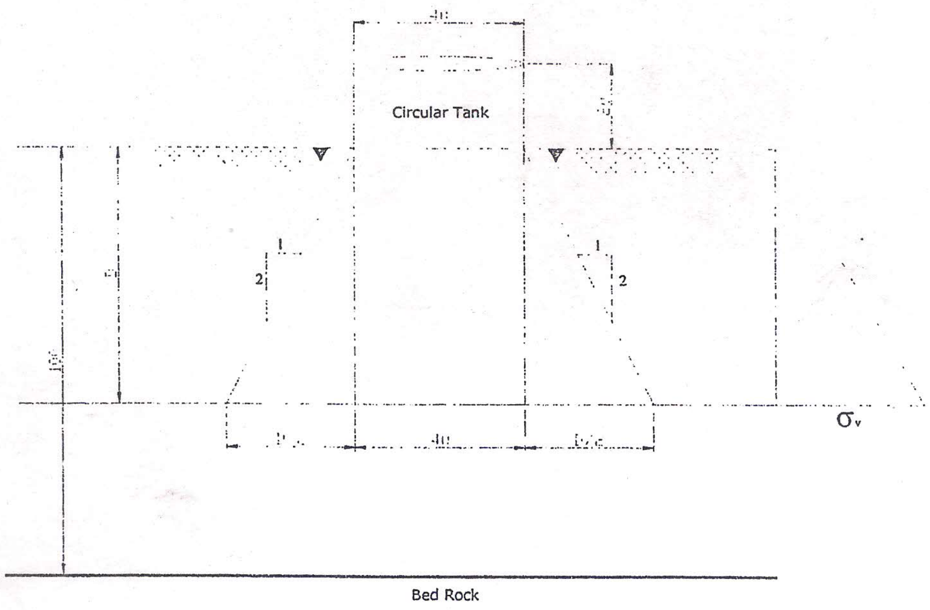
To best understand this problem; students must be familiar with:

1. Phase diagram relationships (Chapter II, Soil Mechanics)
2. Effective stress estimation (Chapter VI, Soil Mechanics)
3. Vertical stress increase estimation (Chapter VII, Soil Mechanics)
4. Settlement of clayey soil beneath foundations (Chapter VIII, Soil Mechanics)

Problem # (2)

Site investigation is to be made for a 40-m diameter and 20-m high water tank. The tank will be built on a thick clay layer. Depth of rock is about 100-m while the water table is at the ground surface. What is the approximate depth of boreholes?

Saturated unit weight (γ_{sat}) of soil is 17-kN/m^3.



Solution:

Let D_1 & D_2 to be the assumed depths of the borehole

q = *The pressure caused by the weight of water*

$q = H \times \gamma_w$

$\gamma_w = 10\ kN/m^3$

Then; $q = 20 \times 10 = 200\ kN/m^2$

Depth of the borehole is the smallest of D_1 or D_2, where;

D_1 : *Is the depth at which* $\Delta\sigma = \frac{1}{10}q$

$q = 200\ kN/m^2$

$\Delta\sigma = \frac{Weight}{Area}$; *the area will be calculated besed on 2-vertical:1-horizontal concept*

$$Then;\ \Delta\sigma = \frac{\frac{\pi}{4} \times (40^2) \times 20 \times 10}{\frac{\pi}{4} \times (40 + D_1)^2} = \frac{3.2 \times 10^5}{(40 + D_1)^2}$$

$$\Rightarrow \frac{3.2 \times 10^5}{(40 + D_1)^2} = \frac{1}{10} \times 200 = 20$$

$\Rightarrow D_1^2 + 80D_1 - 14400 = 0.0$

$D_1 = 86.49m$

D_2: *Is the depth at which* $\Delta\sigma = \frac{5}{100}\sigma'_v$

$\sigma'_v = \gamma' \times H = (\gamma_{sat} - \gamma_w) \times D_2 = (17 - 10) \times D_2 = 7D_2\ kN/m^2$

$\Rightarrow \frac{3.2 \times 10^5}{(40 + D_2)^2} = \frac{5}{100} \times 7D_2$

$\Rightarrow D_2^3 + 80D_2^2 + 1600D_2 - 914285.7 = 0.0$

By Trial & Error Technique :

$D_2 = 72.38m$

Depth of borehole will be $72.38m$

Pro' lcm # (3)

The borehole shown is in a silty soil.

Given that;

$h_w + h_o = 12.24m$

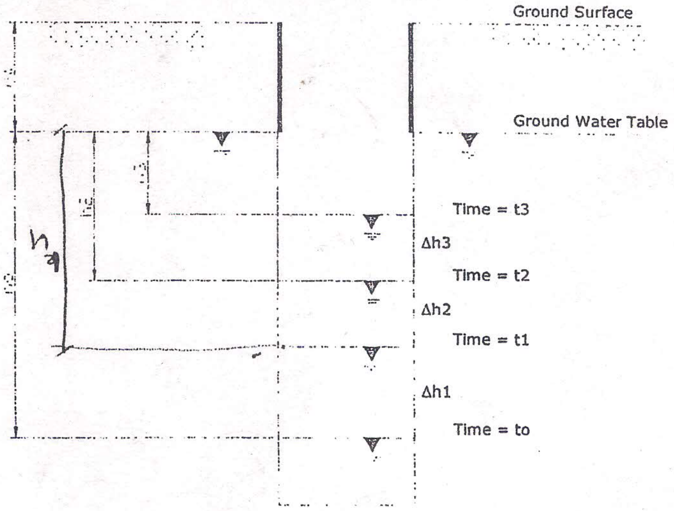
$t_1 = 24hr$

$t_2 = 48hr$

$\Delta h_1 = 1.2m$

$\Delta h_2 = 0.86m$

$\Delta h_3 = 0.60m$



Determine:

1. (h_w) the depth of water table.
2. Determine h_2
3. Determine h_3

Solution:

$$h_o = \frac{(\Delta h_1)^2}{\Delta h_1 - \Delta h_2} = \frac{(1.2)^2}{1.2 - 0.86} = 4.24m$$

$$h_2 = \frac{(\Delta h_2)^2}{\Delta h_1 - \Delta h_2} = \frac{(0.86)^2}{1.2 - 0.86} = 2.175m$$

$$h_3 = \frac{(\Delta h_3)^2}{\Delta h_2 - \Delta h_3} = \frac{(0.6)^2}{0.86 - 0.60} = 1.385m$$

$h_w + h_o = 12.24m$ (*Given*)

$\Rightarrow h_w = 12.24 - 4.24 = 8m$

Problem # (3)

The borehole shown is in a silty soil.

Given that;

$h_w + h_o = 12.24m$

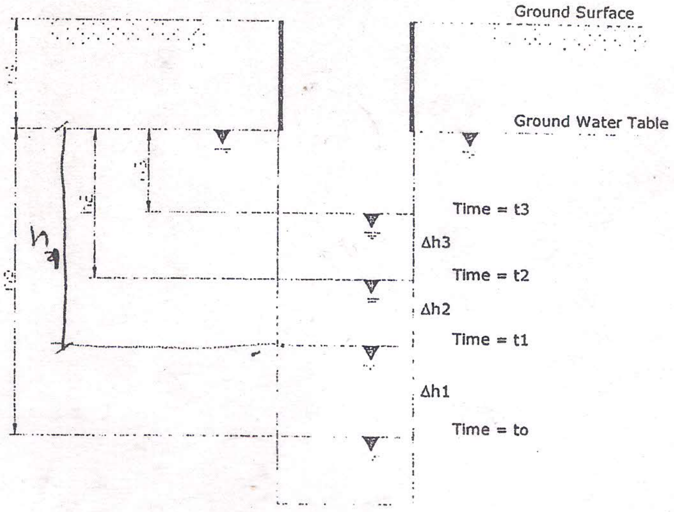
$t_1 = 24hr$

$t_2 = 48hr$

$\Delta h_1 = 1.2m$

$\Delta h_2 = 0.86m$

$\Delta h_3 = 0.60m$



Determine:

1. (h_w) the depth of water table.
2. Determine h_2
3. Determine h_3

Solution:

$$h_o = \frac{(\Delta h_1)^2}{\Delta h_1 - \Delta h_2} = \frac{(1.2)^2}{1.2 - 0.86} = 4.24m$$

$$h_2 = \frac{(\Delta h_2)^2}{\Delta h_1 - \Delta h_2} = \frac{(0.86)^2}{1.2 - 0.86} = 2.175m$$

$$h_3 = \frac{(\Delta h_3)^2}{\Delta h_2 - \Delta h_3} = \frac{(0.6)^2}{0.86 - 0.60} = 1.385m$$

$h_w + h_o = 12.24m$ (*Given*)

$\Rightarrow h_w = 12.24 - 4.24 = 8m$

Problem # (4)

For the soil profile shown below; standard penetration numbers in a clay soil layer were given. Determine and plot the variation of C_u and OCR with depth.

Useful Formulas:

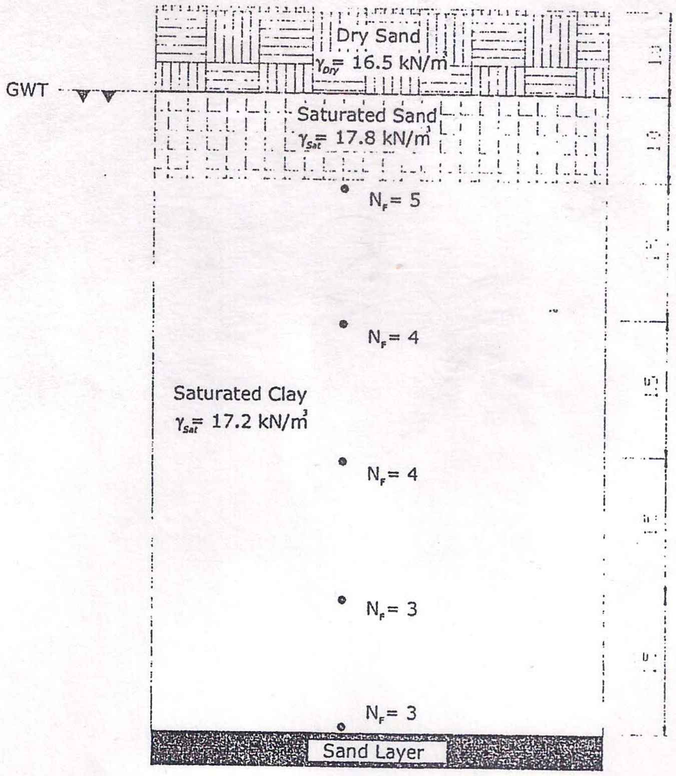
$$C_u = 29N_F^{\;0.72}$$

$$OCR = 0.193\left(\frac{N_F}{\sigma'_v}\right)^{0.689}$$

Notes;

σ'_v *in* MN/m^2

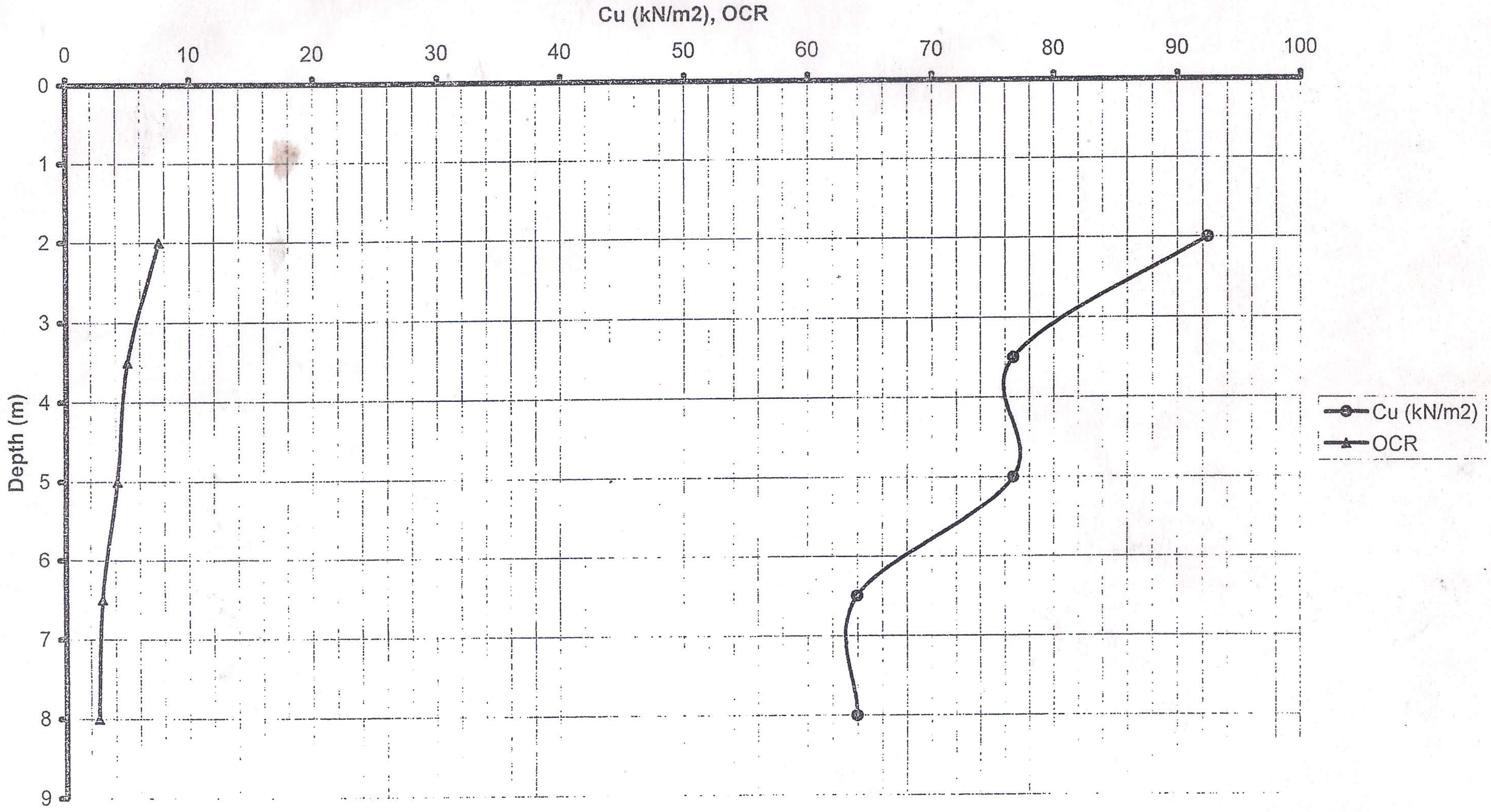
C_u *in* kN/m^2



Solution:

Depth (m)	σ'_v (kN/m²)	N_F	C_u (kN/m²)	OCR
2.0	$16.5 + (17.8 - 9.81) * 1.0 = 24.49$	5	$29 \times (5)^{0.72} = 92.4$	$0.193(\frac{5 \times 1000}{24.3})^{0.689} = 7.54$
3.5	$24.3 + (17.2 - 9.81) * 1.5 = 35.6$	4	$29 \times (4)^{0.72} = 78.68$	$0.193(\frac{4 \times 1000}{35.6})^{0.689} = 5$
5.0	$35.6 + (17.2 - 9.81) * 1.5 = 46.7$	4	$29 \times (4)^{0.72} = 78.68$	$0.193(\frac{4 \times 1000}{46.7})^{0.689} = 4.15$
6.5	$46.7 + (17.2 - 9.81) * 1.5 = 57.8$	3	$29 \times (3)^{0.72} = 63.96$	$0.193(\frac{3 \times 1000}{57.8})^{0.689} = 2.93$
8.0	$57.8 + (17.2 - 9.81) * 1.5 = 68.9$	3	$29 \times (3)^{0.72} = 63.96$	$0.193(\frac{3 \times 1000}{68.9})^{0.689} = 2.6$

Variation of Cu and OCR with depth



CONVERSION TABLE

Force:

1 lb = 4.4482 N

1 U.S. ton = 8.896 kN

I lb/ft = 14.593 N/m (Force per unit length)

1 N = 0.2248 lb

Stress:

1 lb/ft^2 = 47.88 N/m^2

1 U.S. ton/ft^2 = 95.76 kN/m^2

1 kip/ft^2 = 47.88 kN/m^2

1 lb/in^2 = 6.895 kN/m^2

Unit weight:

1 lb/ft^3 = 0.1572 kN/m^3

1 lb/in^3 = 271.43 kN/m^3

Length:

1 ft = 30.48 cm

1 in = 25.4 mm

1 ft = 12 in

1 mi = 5280 ft

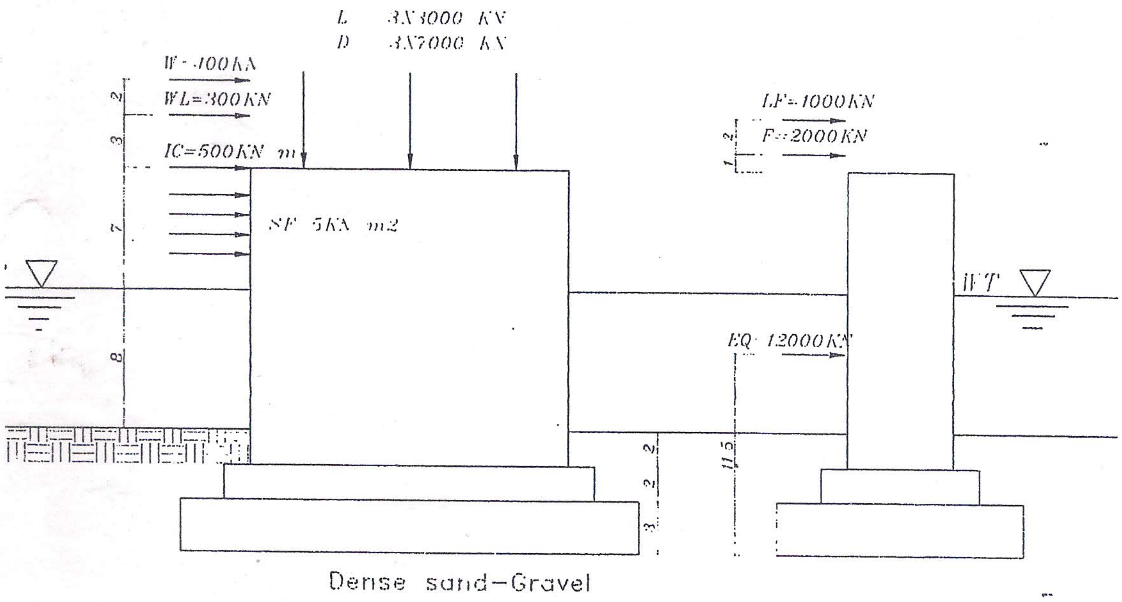
1 mi = 1.609 km

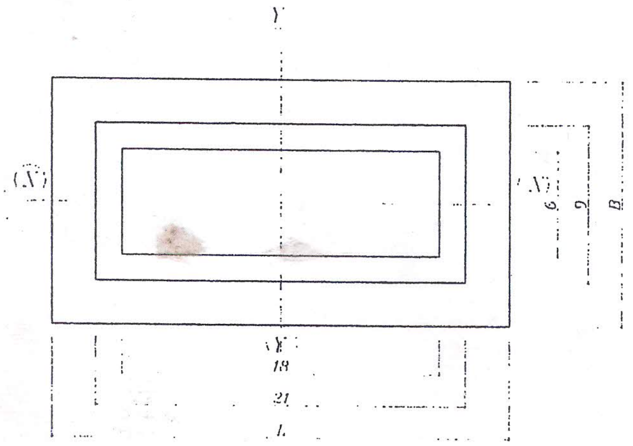
Problem # (5) X

A solid concrete bridge pier is to be constructed on very dense sand – gravel deposit .Determine the width and length of the footing.

- Gross allowable bearing value of soil (allowable base pressure) = 300kN/m^2
- Unit weight of concrete = 24 kN/m^3
- Saturated unit weight of soil = 20 kN/m^3
- Unit weight of water = 10 kN/m^3

Use AASHTO loading groups.
No tension is permitted at any point of the base under any group loading.





Loads:

D : Dead load

L : Live load

W: Wind load

IC: Ice thrust

SF: Stream Flow Pressure

LF: Longitudinal Forces (Braking Forces)

F: Longitudinal Forces due to friction

EQ: Earthquakes

Load Groups:

Group 1: DL + SF + LL

Group 2: DL + SF + W

Group 3: DL + SF + EQ

Group 4: DL + SF + LL + IC

Group 5: DL + SF + W + IC

Group 6: DL + SF + LL + LF + F + WL + 0.3W

Solution:

Assume B = 14 m, L = 26 m

Calculations of Loadings (Vertical & Horizontal forces):

- Total dead load (DL) = weight of pier + weight of footing + weight of soil + dead load from the superstructure

  - Weight of pier = (6×18×7×24) + ((6×18×10) × (24-10)) + (9×21×2× (24-10))

    = 38,556 kN

  - Weight of footing = 26×14×3× (24-10) = 15,288 kN

  - Weight of soil = (26×14-21×9) ×4× (20-10) + (21×9-18×6) ×2× (20-10)

    = 8,620 kN

  - Dead load from the superstructure = 3×7000 = 21,000 kN

∴ Total dead load (DL) = 83,464 kN

- Total live load (LL) = 3×3000 = 9000 kN
- Ice thrust force (IC) = 500×6 = 3000 kN
- Stream flow force (SF) = 5 ×6×8 = 240 kN

Calculations of Moments (M_x & M_y):

Load Type	Vertical Force (kN)	Horizontal Force (kN)	Moment Arm (m)	M_y (kN.m)	M_x (kN.m)
DL	83464	-	0	-	-
LL	9000	-	0	-	-
SF	-	240	11	2640	-
W	-	400	27	10800	-
0.3 W	-	120	27	3240	-
WL	-	300	25	7500	-
IC	-	3000	15	45000	-
LF	-	1000	25	-	25000
F	-	2000	23	-	46000
EQ	-	120000	11.5	-	138000

$$q = \frac{Q}{BL}\left(1 \pm \frac{6e_x}{L} \pm \frac{6e_y}{B}\right)$$

Group 1 : DL + SF + LL

$Q = 92,464\ kN$

$M_y = 2,640\ kN.m$

$$e_x = \frac{M_y}{Q} = \frac{2,640}{92,464} = 0.029\ m$$

$$q_{\min}^{\max} = \frac{Q}{BL}\left(1 \pm \frac{6e_x}{L}\right) = \frac{92,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.029}{26}\right) = \begin{matrix} 255.7\ kN/m^2 \\ 252.3\ kN/m^2 \end{matrix} < 300\ kN/m^2$$

Group 2 : DL + SF + W

$Q = 83,464\ kN$

$M_y = 2,640 + 10,800 = 13,440\ kN.m$

$$e_x = \frac{M_y}{Q} = \frac{13,440}{83,464} = 0.16\,m$$

$$q_{\min}^{\max} = \frac{83,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.16}{26}\right) = \begin{matrix} 237.8\,kN/m^2 \\ 220.8\,kN/m^2 \end{matrix} < 300\ kN/m^2$$

Group 3 : DL + SF + EQ **(F.S = 1.35)**

$Q = 83,464\ kN$

$$M_y = 2,640\ kN.m \Rightarrow e_x = \frac{M_y}{Q} = \frac{2,640}{83,464} = 0.032\ m$$

$$M_x = 138,000\ kN.m \Rightarrow e_y = \frac{M_x}{Q} = \frac{138,000}{83,464} = 1.65\,m$$

$$q_{\min}^{\max} = \frac{1}{F.S}\frac{Q}{BL}\left(1 \pm \frac{6e_x}{L} \pm \frac{6e_y}{B}\right) = \frac{1}{1.35} \times \frac{83,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.032}{26} \pm \frac{6 \times 1.65}{14}\right) = \begin{matrix} 291\,kN/m^2 \\ 48.6\,kN/m^2 \end{matrix} < 300\ kN/m^2$$

Group 4 : DL + SF + LL + IC **(F.S = 1.4)**

$Q = 92,464\ kN$

$$M_y = 2,640 + 45,000 = 47,640\ kN.m \Rightarrow e_x = \frac{M_y}{Q} = \frac{47,640}{92,464} = 0.52\,m$$

$$q_{\min}^{\max} = \frac{1}{F.S}\frac{Q}{BL}\left(1 \pm \frac{6e_x}{L}\right) = \frac{1}{1.4} \times \frac{92,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.52}{26}\right) = \begin{matrix} 203.2\,kN/m^2 \\ 159.7\,kN/m^2 \end{matrix} < 300\ kN/m^2$$

Group 5 : DL + SF + W + IC **(F.S = 1.5)**

$Q = 83,464\ kN$

$$M_y = 2,640 + 10,800 + 45,000 = 58,440\ kN.m \Rightarrow e_x = \frac{M_y}{Q} = \frac{58,440}{83,464} = 0.7\,m$$

$$q_{\min}^{\max} = \frac{1}{F.S}\frac{Q}{BL}\left(1 \pm \frac{6e_x}{L}\right) = \frac{1}{1.5} \times \frac{83,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.7}{26}\right) = \begin{matrix} 177.6\,kN/m^2 \\ 128.2\,kN/m^2 \end{matrix} < 300\ kN/m^2$$

Group 6 : DL + SF + LL + LF + F + WL + 0.3W (F.S = 1.25)

$Q = 92,464\ kN$

$$M_y = 2,640 + 7,500 + 3,240 = 13,380\ kN.m \Rightarrow e_x = \frac{M_y}{Q} = \frac{13,380}{92,464} = 0.14\ m$$

$$M_x = 25,000 + 46,000 = 71,000\ kN.m \Rightarrow e_y = \frac{M_x}{Q} = \frac{71,000}{92,464} = 0.77\ m$$

$$q_{\min}^{\max} = \frac{1}{1.25} \times \frac{92,464}{14 \times 26}\left(1 \pm \frac{6 \times 0.14}{26} \pm \frac{6 \times 0.77}{14}\right) = \begin{matrix} 276.8\ kN/m^2 \\ 129.6\ kN/m^2 \end{matrix} < 300\ kN/m^2$$

So, dimensions of the pier footing are perfectly alright for both structural safety and economical point of view.

Problem # (6) Ch 3

For a foundation with two way eccentricity; the soil properties are: γ = 18 kN/m^3, ϕ=35° and c=0. The design criteria are D_f =1 m, B=1.5 m, L=2 m, e_B = 0.3 m and e_L = 0.364 m. Determine the gross ultimate load that the foundation could carry.

Solution:

$$\frac{e_B}{B} = \frac{0.3}{1.5} = 0.2 > \frac{1}{6}\ ,\ \frac{e_L}{L} = \frac{0.364}{4} = 0.182 > \frac{1}{6} \Rightarrow case\ (I)$$

$$B_1 = B\left(1.5 - \frac{3e_B}{B}\right) = 1.5\left(1.5 - \frac{3\times 0.3}{1.5}\right) = 1.35\,m$$

$$L_1 = L\left(1.5 - \frac{3e_L}{L}\right) = 2\left(1.5 - \frac{3\times 0.364}{2}\right) = 1.908\,m$$

Let $L' = L_1 = 1.908$

$$\therefore A' = \frac{1}{2}B_1L_1 = \frac{1}{2}\times 1.35\times 1.908 = 1.29\,m^2 \Rightarrow B' = \frac{A'}{L'} = \frac{1.29}{1.908} = 0.675\,m$$

$$q_u = qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}$$

From table (3.4) : $\phi = 35°$, $N_q = 33.3$, $N_\gamma = 48.03$

$$F_{qs} = 1 + \frac{B'}{L'}\tan\phi = 1 + \frac{0.675}{1.908}\tan 35 = 1.248$$

$$F_{qd} = 1 + \left[2\tan\phi(1-\sin\phi)^2\right]\frac{D_f}{B} = 1 + \left[2\tan 35(1-\sin 35)^2\right]\frac{1}{1.5} = 1.167$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{0.675}{1.908}\right) = 0.858$$

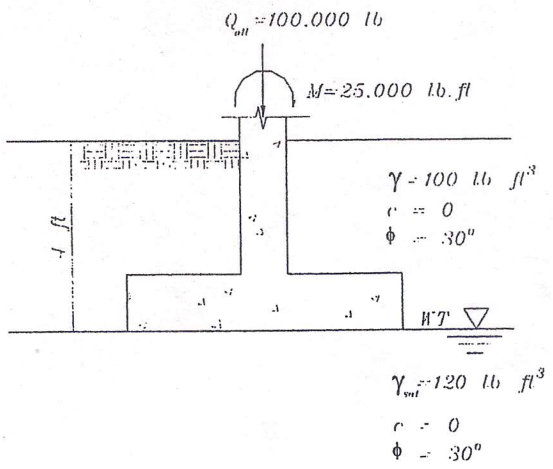
$$F_{\gamma d} = 1$$

$$q_u = (18\times 1\times 33.3\times 1.248\times 1.167) + \left(\frac{1}{2}\times 18\times 0.675\times 48.03\times 0.858\times 1\right) = 1123.32\,kN/m^2$$

$$Q_u = q_u \times A' = 1123.32\times 1.29 = 1449.1\,kN$$

Problem # (7) ch3

A square footing is shown in figure below. Use a F.S of 6 and determine the size of the footing.



Solution:

$$e = \frac{M}{Q} = \frac{25,000}{100,000} = 0.25\ ft$$

$$B' = B - 2e = B - 0.5\ ,\ L' = L = B$$

From table (3.4) : $\phi = 30^\circ$, $N_q = 18.4$, $N_\gamma = 22.4$

$$F_{qs} = 1 + \frac{B'}{L'}\tan\phi = 1 + \frac{B-0.5}{B}\tan 30 = 1.58 - \frac{0.29}{B}$$

$$F_{qd} = 1 + \left[2\tan\phi(1-\sin\phi)^2\right]\frac{D_f}{B} = 1 + \left[2\tan 30(1-\sin 30)^2\right]\frac{4}{B} = 1 + \frac{1.15}{B}$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{B-0.5}{B}\right) = 0.6 + \frac{0.2}{B}$$

$$F_{\gamma d} = 1$$

$$q_u = qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}$$

$$q_u = \left[(100\times 4)\times 18.4\left(1.58 - \frac{0.29}{B}\right)\left(1 + \frac{1.15}{B}\right) + \frac{1}{2}\times(120 - 62.4)\times(B-0.5)\times 22.4\times\left(0.6 + \frac{0.2}{B}\right)\right]$$

But $q_u = q_{all} \times F.S$

$$q_u = \frac{100,000}{(B-0.5)B}\times 6 = 122.67\left(1.58 - \frac{0.29}{B}\right)\left(1 + \frac{1.15}{B}\right) + 107.52\times(B-0.5)\left(0.6 + \frac{0.2}{B}\right)$$

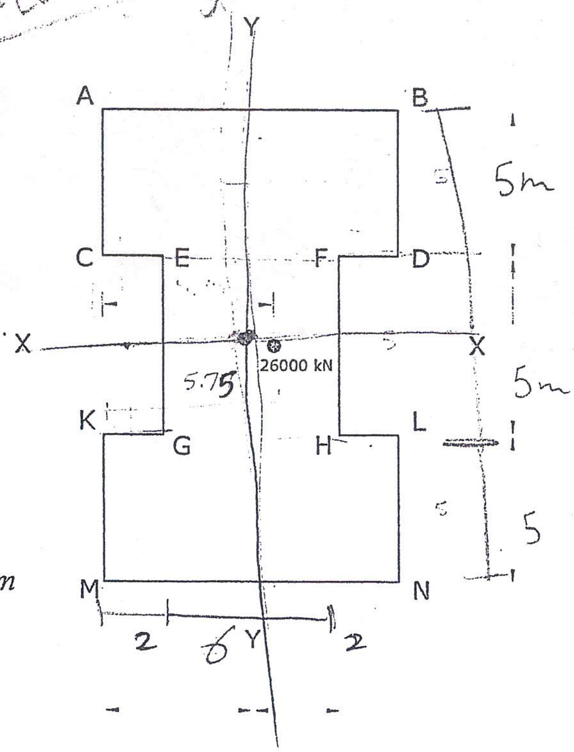
By trial and error : $B \cong 6.5\ ft$

Problem # (8)

Calculate the base pressure at the points indicated below the mat (raft) foundation shown. Weight of the mat will not be included.

Total vertical load acting on the foundation is 26000 kN.

Solution:



$$A = 10\times 15 - 2\times(2\times 5) = 130\ m^2$$

$$I_{yy} = 2\times\left(\frac{10^3\times 5}{12}\right) + \left(\frac{6^3\times 5}{12}\right) = 923.33\ m^4$$

$$I_{xx} = \left(\frac{10\times 15^3}{12}\right) - 2\times\left(\frac{2\times 5^3}{12}\right) = 2770.83\ m^4$$

$$q = \frac{\Sigma Q}{A} \pm \frac{M_x Y}{I_x} \pm \frac{M_y X}{I_y}$$

$$e_x = 5.75 - 5 = 0.75\ m$$

$$e_y = 0.0\ m$$

$$M_x = \sum Q\times e_y \Rightarrow M_x = 26000\times 0.0 = 0.0$$

$$M_y = \sum Q\times e_x \Rightarrow M_y = 26000\times 0.75 = 19500\ kN.m$$

Then;

$$q = \frac{26000}{130} \pm \frac{19500\times X}{923.33} = 200 \pm 21.12X$$

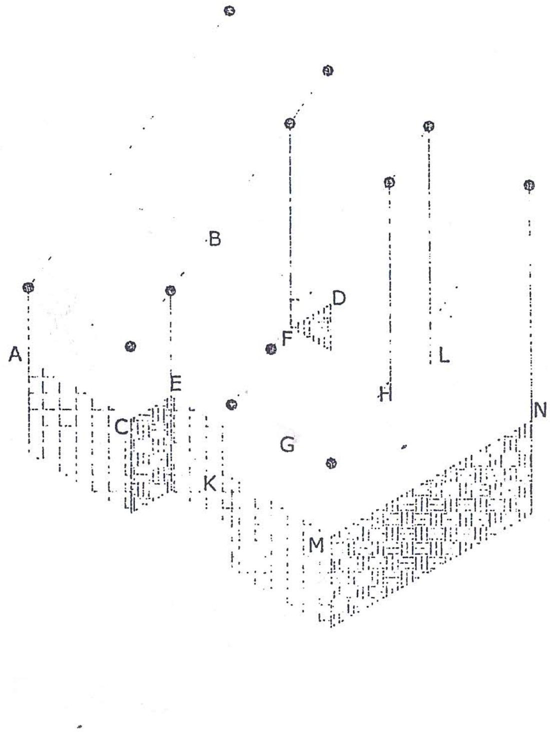
$$q_{(A,C,K,M)} = 200 - 21.12\times 5 = 94.4\ kN/m^2$$

$$q_{(E,G)} = 200 - 21.12\times 3 = 136.64\ kN/m^2$$

$$q_{(F,H)} = 200 + 21.12\times 3 = 263.36\ kN/m^2$$

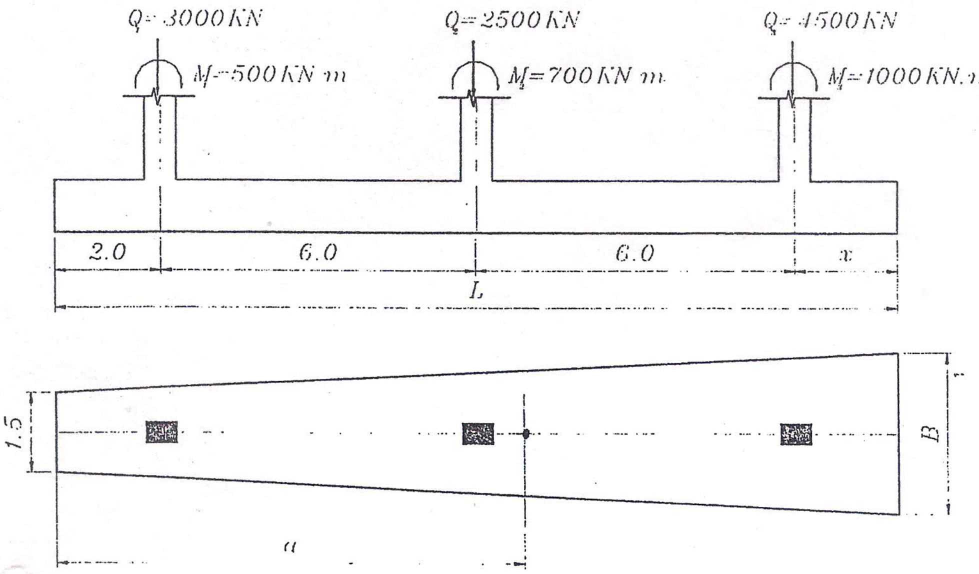
$$q_{(B,D,L,N)} = 200 + 21.12\times 5 = 305.6\ kN/m^2$$

This figure shows the pressure distribution below the mat foundation based on the assumption that the pressure distribution is linear.

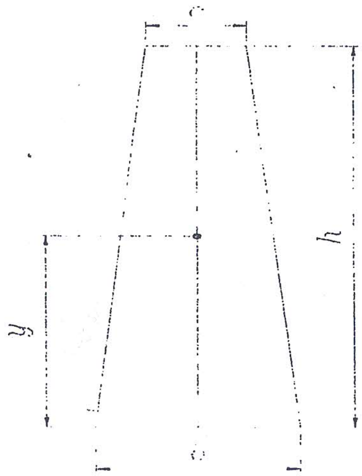


Problem # (9)

Consider the weight of footing and determine L and B of a trapezoidal footing for a uniform soil pressure of 300 kN/m^2, thickness of the footing is 1.0 m



__Useful formula:__



$$y = \frac{h}{3}\left(\frac{2c+b}{c+b}\right)$$

__Solution:__

$$\text{Weight of footing} = \left(\frac{1.5+B}{2}\right) \times L \times 24 = 12\,L(1.5+B)$$

$$\sum Q = 0$$

$$3000 + 2500 + 4500 + 12\,L(1.5+B) = 300\left(\frac{1.5+B}{2} \times L\right)$$

$$10000 + 12\,L(1.5+B) = 150\,L(1.5+B)$$

$Let\ (1.5+B)=Y$

$10000+12\,LY=150\,LY$

$10000=138\,LY$

$LY=72.5$ ....(1)

$\sum M=0$ (Moment taken about the centroid of the base)

$3000\,(a-2)+500+2500\,(a-8)+700-4500\,(14-a)+1000=0$

$100\,a=888 \Rightarrow a=8.88\,m$

But;

$$L-8.88=\frac{L}{3}\left(\frac{2\times1.5+B}{1.5+B}\right)=\frac{L}{3}\left(\frac{1.5+Y}{Y}\right)$$

$3Y(L-8.88)=1.5L+LY$

$3LY-26.64Y=1.5L+LY$

$1.5L=2LY-26.64Y$ ....(2)

Substitute by LY from equation (1) in equation (2)

$1.5L=2\times72.5-26.64Y$

$$1.5L=145-26.64\left(\frac{72.5}{L}\right)$$

$1.5L^2-145L+1931.4=0$

$$L=\frac{145\pm\sqrt{145^2-4\times1.5\times1931.4}}{2\times1.5}=15.95m$$

$$\therefore Y=\frac{72.5}{15.95}=4.54 \Rightarrow B=3.04m$$

Problem # (10)

Ch 5

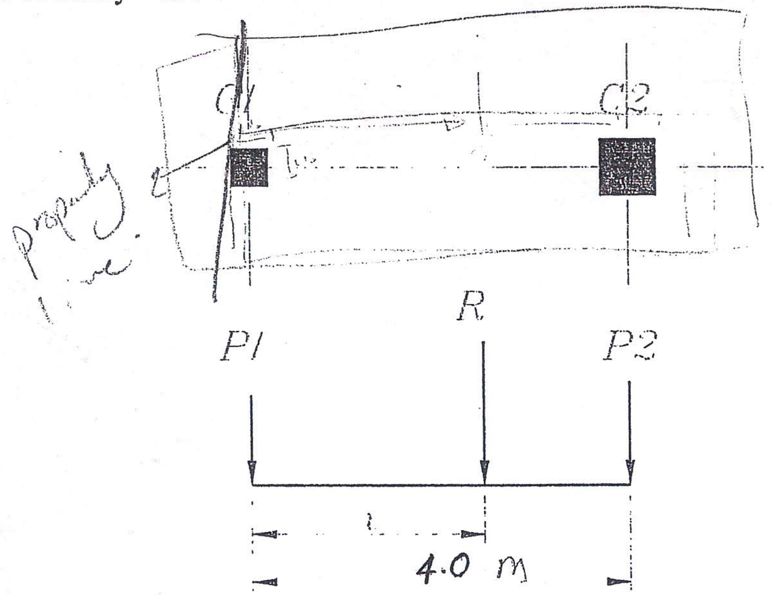
Determine the dimensions of a rectangular footing, given the following data:

Column (1): PD = 55 t, PL = 45 t and 40x40 cross section.

Column (2): PD = 90 t, PL = 70 t and 60x60 cross section.

Ultimate gross soil pressure = 3.7 kg/cm^2, $D_f = 1.5m$ and $\gamma_{soil} = 1.7t/m^3$

Factor of safety = 2.0



Solution:

Assume depth of footing = 60 cm

$$q_{ult(net)} = q_{ult(gross)} - h_s\gamma_s - h_c\gamma_c$$

$$= 37 - 0.9 \times 1.7 - 0.6 \times 2.5 = 33.97 t/m^2$$

$$q_{all(net)} = \frac{q_{ult(net)}}{F.S.} = \frac{33.97}{2} = 16.99$$

$$A_{required} = \frac{\sum Q}{q_{all(net)}} = \frac{55 + 45 + 90 + 70}{16.99} = 15.30 m^2$$

$$R = P_1 + P_2 = (55 + 45) + (90 + 70) = 260 t$$

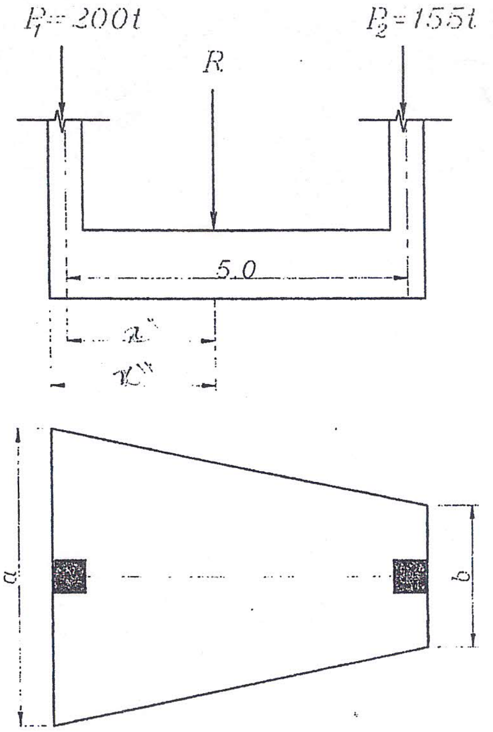
$$260x = (90 + 70) \times 4 \Rightarrow x = 2.46 m$$

$$L = 2(2.46 + 0.2) = 5.32m. \text{ So, it will be taken as } 5.4 \text{ m}$$

$$\therefore B = \frac{A}{L} = \frac{15.30}{5.4} = 2.83m, \text{ and it will be taken as } 2.9 \text{ m}$$

Problem # (11) Ch 5

Determine the dimensions of a trapezoidal footing to support the following loads:
Column (1): PD = 120 t, PL = 80 t and 50x50 cross section.
Column (2): PD = 90 t, PL = 65 t and 50x50 cross section.
Allowable net soil pressure = 2.0 kg/cm^2



Solution:

$155(5) = (155+200)x'$

$x' = 2.18m$

$x'' = 2.18+0.25 = 2.43m$

$$A_{required} = \frac{\sum Q}{q_{all(net)}} = \frac{200+155}{20} = 17.75m^2$$

$$\therefore 17.75 = \left(\frac{a+b}{2}\right)\times 5.5$$

$a+b = 6.45$ .... (1)

But:

$$x'' = \frac{L}{3}\left(\frac{2b+a}{b+a}\right) = \frac{5.5}{3}\left(\frac{2b+a}{b+a}\right)$$ .... (2)

$$2.43 = \frac{5.5}{3}\left(\frac{6.45+b}{6.45}\right)$$

$$1.325 = \frac{6.45+b}{6.45} \Rightarrow 8.55 = 6.45 + b \Rightarrow b = 2.1m$$

$$a = 6.45 - 2.1 = 4.35m$$

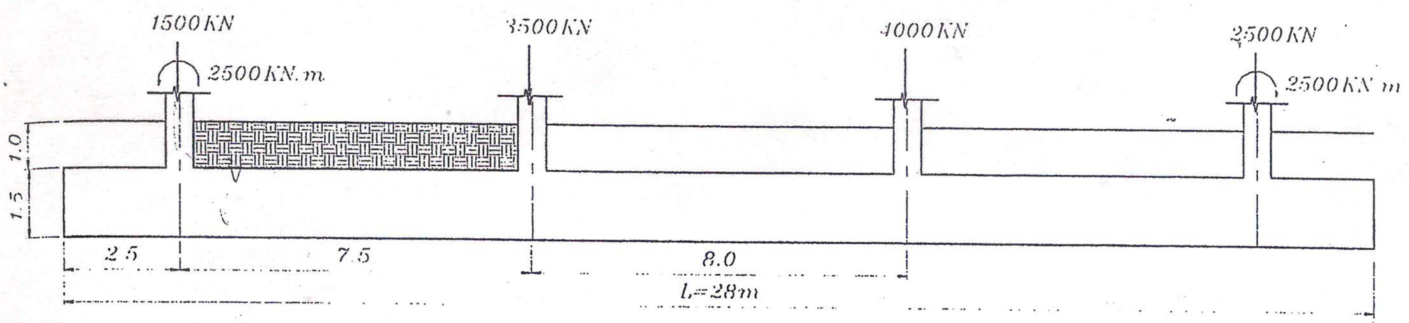
Problem # (12) ch 5 ✓

A combined footing consists of four columns as shown in figure, determine:

1- Width of the combined footing

Allowable gross soil pressure = 255 kN/m^2.

$\gamma_c = 24\,kN/m^3$ and $\gamma_{soil} = 20\,kN/m^3$



Solution:

Assume that the width of the footing is B

Weight of footing & soil = $28 \times B(1.5 \times 24 + 1 \times 20) = 1568\,B$

$\sum Q = 1500 + 3500 + 4000 + 2500 + 1568\,B = (11500 + 1568\,B)kN$

$\sum M$ (Moment taken about the centroid of the base)

$\sum M = -1500 \times 11.5 - 2500 - 3500 \times 4 + 2500 + 2500 \times 11.5 = 13500\ kN.m$

Eccentricity:

$$e = \frac{\sum M}{\sum Q} = \frac{13500}{11500 + 1568\,B}$$

$$q = \frac{Q}{BL}\left(1 \pm \frac{6e}{L}\right)$$

$$q_{max} = \frac{Q}{BL}\left(1 + \frac{6e}{L}\right) \Rightarrow 255 = \frac{11500 + 1568B}{28B}\left(1 + \frac{6 \times 13500}{28(11500 + 1568B)}\right)$$

$$255 = \frac{11500 + 1568\,B}{28\,B}\left(1 + \frac{81000}{322000 + 43904\,B}\right)$$

$$7140\,B = (11500 + 1568\,B)\left(\frac{403000 + 43904\,B}{322000 + 43904\,B}\right)$$

$$244633088\,B^2 + 1162280000\,B - 4634500000 = 0$$

$$B^2 + 4.751B - 18.945 = 0$$

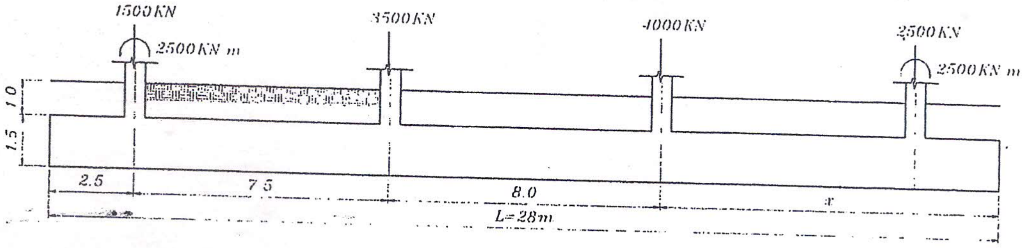
$$\therefore B = \frac{-4.751 \pm \sqrt{4.751^2 - 4\times -18.945}}{2} = \begin{matrix} 2.58m \\ 7.334m \end{matrix}$$

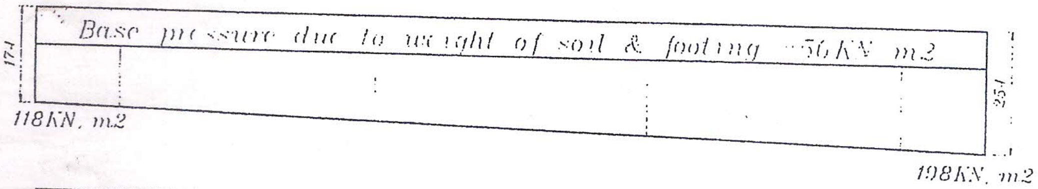
Take B = 2.6 m and check $q_{max} < (q_{all} = 255\ kN/m^2)$

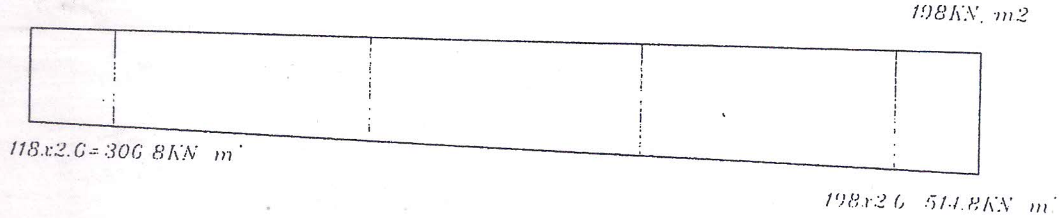
$$q_{max} = \frac{11500+1568\times 2.6}{28\times 2.6}\left(1+\frac{6\times 13500}{28(11500+1568\times 2.6)}\right) = 253.7 < 255 kN/m^2$$

$$q_{min} = \frac{11500+1568\times 2.6}{28\times 2.6}\left(1-\frac{6\times 13500}{28(11500+1568\times 2.6)}\right) = 174 kN/m^2 > 0$$

Base pressure due to weight of soil & footing = 1.5x24 + 1x20 = 56 kN/m²







Problem # (13)

Given the cantilever footing shown below with the following data:
$Q_1 = 1500$ kN, $Q_2 = 2500$ kN, $q_{all} = 200$ kN/m^2.
Design the footing for a uniform soil pressure distribution.
The exterior footing is rectangular ($B_1 \times B_2$) while the interior one is square ($B \times B$).

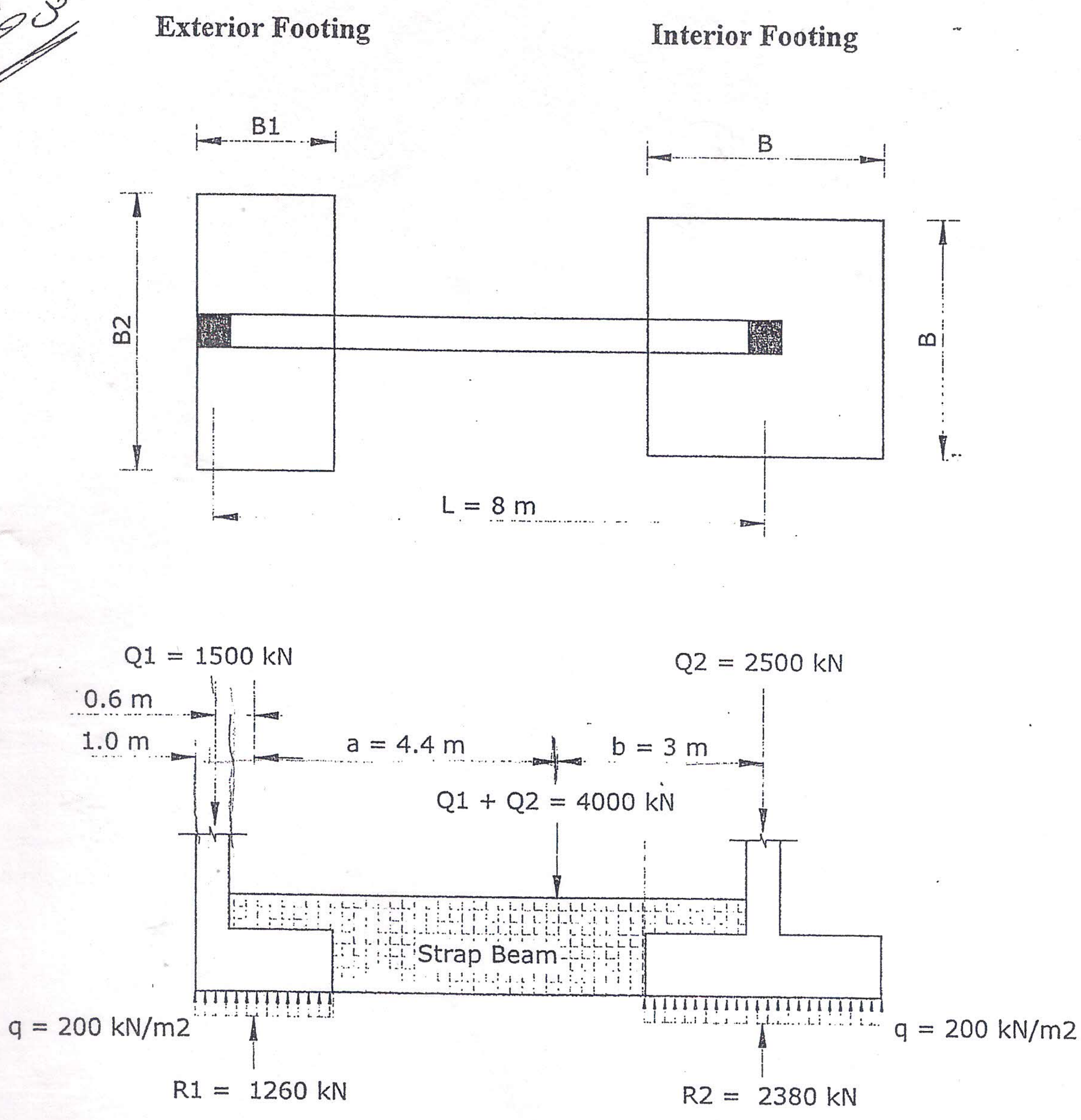


Figure (1)

Solution:

First; locate the point of application of the resultant of the two forces

$\Sigma Q = Q_1 + Q_2 = 1500 + 2500 = 4000\ kN$

$$b = \frac{1500 \times 8}{4000} = 3.0\ m$$

Select $B_1 = 2.0$ m

Then, a = 4.4 m

$$R_1 = \frac{3 \times 4000}{7.4} = 1260\ kN$$

$$R_2 = 4000 - 1260 = 2380\ kN$$

$$Now,\ B_2 = \frac{1260}{200 \times 2} = 4.0\ m$$

$$B = \sqrt{\frac{2380}{200}} = 3.45\ m$$

By referring to Figure (1), you can draw the shear and moment diagrams. Figure (2) outlines the general shape of them, and values are left to student.

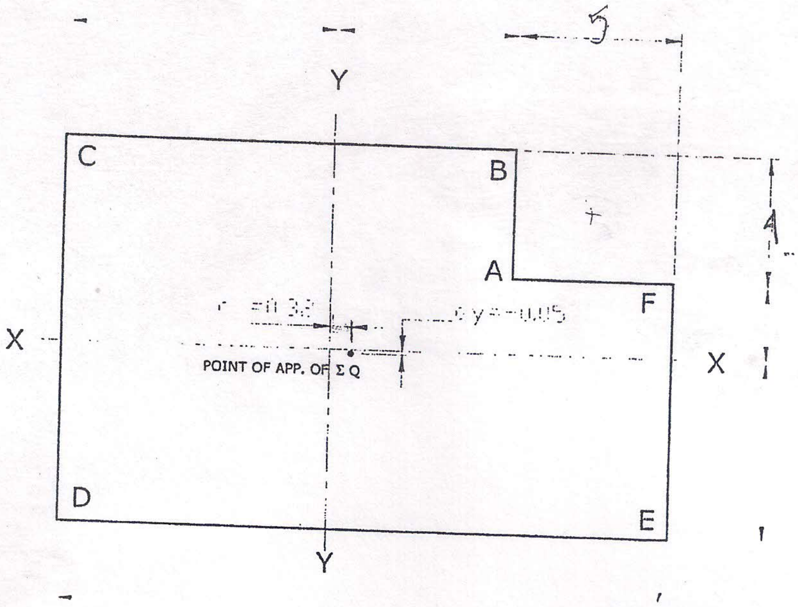
Note, the strap beam does not take soil pressure.



Figure (2)

Problem # (14)

Calculate the base pressure at the points indicated below the mat (raft) foundation shown. Height of the mat is 2.0 m and $\gamma_{con} = 24$ kN/m^3



X-X & Y-Y ARE CENTROIDAL AXES

Solution:

$$A = 18\times12 - 5\times4 = 196\ m^2$$

$$\bar{X} = \frac{18\times12\times9 - 4\times5\times(2.5+13)}{196} = \frac{1634}{196} = 8.34m$$

$$\bar{Y} = \frac{18\times12\times6 - 4\times5\times(8+2)}{196} = \frac{1096}{196} = 5.59m$$

$$I_{yy} = \left(\frac{18^3\times12}{12} + 12\times18\times(0.66)^2\right) - \left(\frac{5^3\times4}{12} + 20(2.5+4.66)^2\right) = 4859\ m^4$$

$$I_{xx} = \left(\frac{18\times12^3}{12} + 12\times18\times(0.41)^2\right) - \left(\frac{5\times4^3}{12} + 4\times5\times(2+2.41)^2\right) = 2212\ m^4$$

$$I_{XY} = 18\times12\times(0.66)\times(0.41) - 20\times(4.41)\times(2.5+4.66) = -573m^4$$

$$\Sigma Q = 9\times1200 + 2\times1800 + (12\times18 - 5\times4)\times24\times2 = 23808\ kN$$

{Column Loads} + {Own wt. of footing}

Moment about the left side :

$$3\times1200\times[2+6.5+11]+2\times1800\times16=127800\ kN.m$$
$$12\times18\times24\times2\times9=93312\ kN.m$$
$$-5\times4\times24\times2\times15.5=-14880\ kN.m$$
$$\sum M=206232\ kN.m$$

$$X=\frac{206232}{23808}=8.66m\Rightarrow e_X=0.32m$$

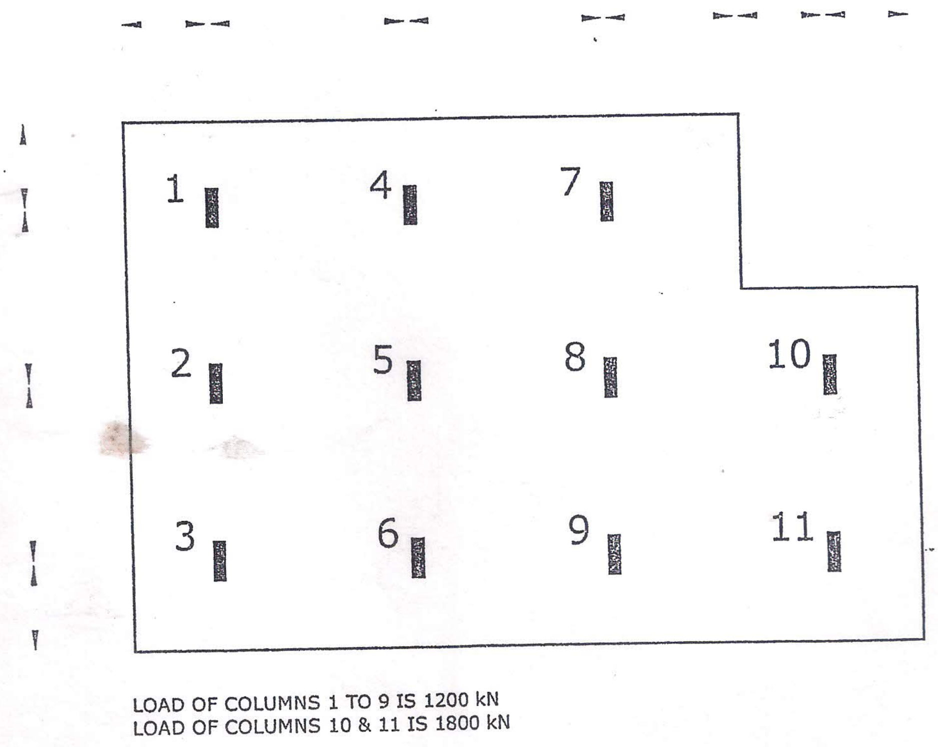
$Note; e_X=8.66-8.34=0.32m$

Moment about the bottom line:

$$3\times1200\times[10+6+2]+1800\times[6+2]=79.2\ kN.m$$
$$12\times18\times24\times2\times6=62.208\ kN.m$$
$$-5\times4\times24\times2\times10=-9.60\ kN.m$$
$$\sum M=131.808\ kN.m$$

$$Y=\frac{131.808}{23808}=5.54m\Rightarrow e_Y=0.05m$$

$Note; e_Y=5.54-5.59=-0.05m$



$$M_X = \Sigma Q \times e_y = 23808 \times (-0.05) = -1190.4\ kN.m$$
$$M_Y = \Sigma Q \times e_X = 23808 \times (0.32) = 7618.56\ kN.m$$

$$q = \frac{\Sigma Q}{A} \pm \left(\frac{M_Y I_X - M_X I_{XY}}{I_X I_Y - (I_{XY})^2} \right) X \pm \left(\frac{M_X I_Y - M_Y I_{XY}}{I_X I_Y - (I_{XY})^2} \right) Y$$

$$\Rightarrow q = \frac{23808}{196} \pm \left(\frac{7618.56 \times 2212 - (-1190.4) \times (-573)}{4860 \times 2212 - (-573)^2} \right) X$$
$$\pm \left(\frac{(-1190.4) \times 4859 - (7618.56) \times (-573)}{4860 \times 2212 - (-573)^2} \right) Y$$

$$\Rightarrow q = 121.47 \pm 1.55X \pm 0.136Y$$

$$q_A = 121.47 + 1.55 \times 4.66 - 0.136 \times 2.41 = 128.36\ kN/m^2$$
$$q_B = 121.47 + 1.55 \times 4.66 - 0.136 \times 6.41 = 127.82\ kN/m^2$$
$$q_C = 121.47 - 1.55 \times 8.34 - 0.136 \times 6.41 = 107.67\ kN/m^2$$
$$q_D = 121.47 - 1.55 \times 8.34 + 0.136 \times 5.59 = 109.3\ kN/m^2$$
$$q_E = 121.47 + 1.55 \times 9.66 + 0.136 \times 5.59 = 137.2\ kN/m^2$$
$$q_F = 121.47 + 1.55 \times 9.66 - 0.136 \times 2.41 = 136.11\ kN/m^2$$

Retaining Walls

A. Useful Relationships:

$$K_a = \tan^2(45 - \Phi/2)$$

$$K_p = \tan^2(45 + \Phi/2)$$

Where;

K_a is Rankine active earth pressure coefficient

K_p is Rankine passive earth pressure coefficient

ϕ is the angle of internal friction of soil

Note; $K_p = \frac{1}{K_a}$

$$FS_{(Overturning)} = \frac{\sum M_R}{\sum M_O}$$

Where;

M_O = sum of the moments of forces tending to overturn the wall

M_R = sum of the moments of forces tending to resist overturning of the wall

Note; FS against overturning is to be > 2

$$FS_{(Sliding)} = \frac{\sum F_R}{\sum F_d}$$ Where;

F_R = sum of the horizontal resisting forces

F_d = sum of the horizontal driving forces

Notes;

- *FS against sliding is to be > 1.5*
- *If the factor of safety against sliding < 1.5, dimensions of the wall is to be changed or a key is needed*

For bearing capacity check; the value of q_{max} (q_{toe}) is to be less than the allowable soil pressure (q_{all}) and no tension is permitted under the base of the wall. The factor of safety against bearing capacity failure will be:

$$FS_{(Bearing\ Capacity)} = \frac{q_{ult}}{q_{toe}}$$

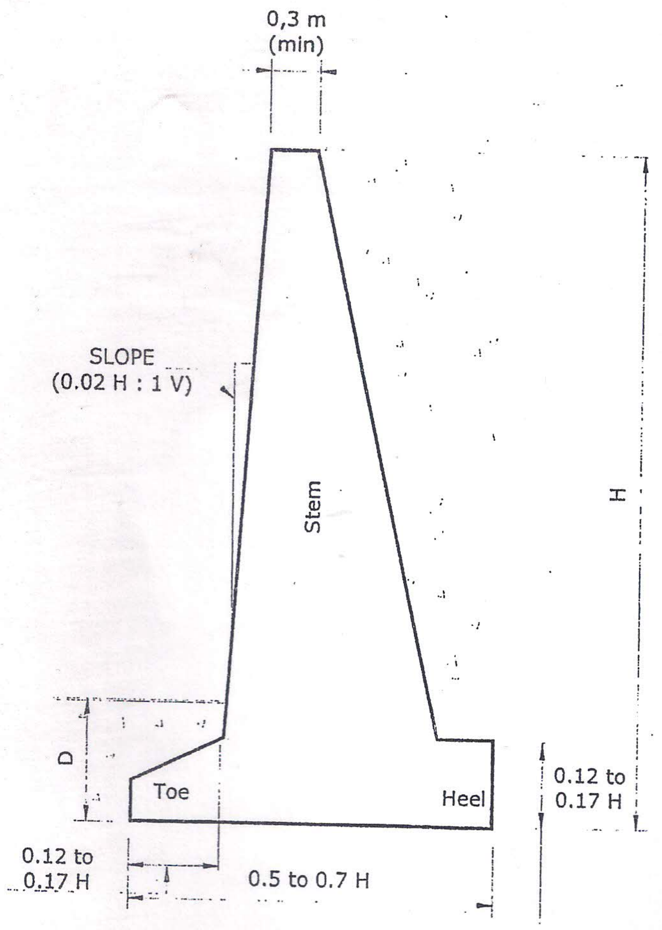
Notes;

- q_{ult} is determined from the Terzaghi's general bearing capacity equation

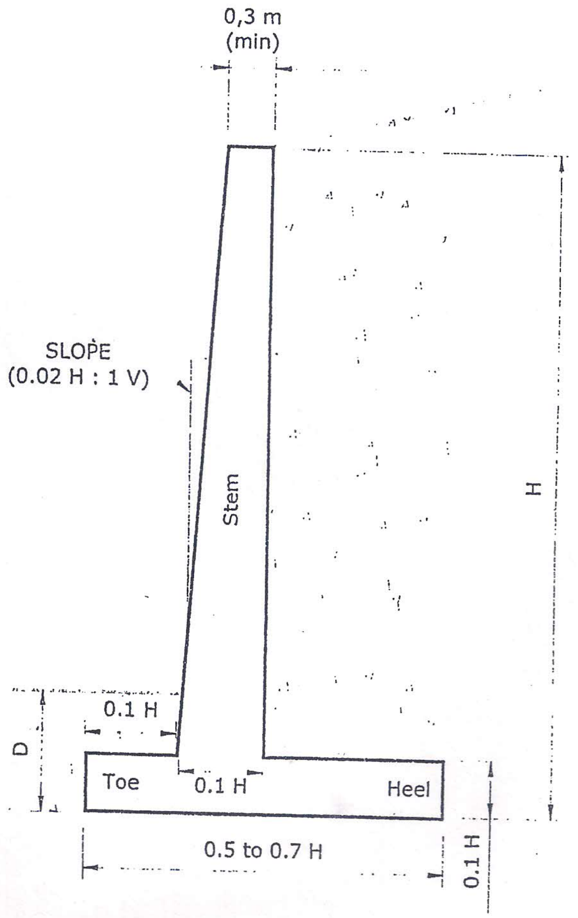
$$q_u = C\,N_c\,F_{ci}\,F_{cd} + q\,N_q\,F_{qi}\,F_{qd} + \frac{1}{2}\gamma\,B\,N_\gamma\,F_{\gamma i}\,F_{\gamma d}$$

- *FS against bearing capacity failure is to be > 3*

B. Approximate Dimensions for Various Components of Retaining Wall for Initial Stability Checks:



D min = 0.6 m
Gravity Retaining Wall



D min = 0.6 m
Cantilever Retaining Wall

Problem # (15)

This is a nice chance for you – as a student of Civil Engineering – to design a cantilever retaining wall since it is the first one in retaining structures.

The following points will be taken in consideration:

- The passive force of the soil will be neglected in the design since the soil in front of the wall will be excavated in the future due to water pipe installation.
- No water pressure will be considered in the backfill since an adequate drainage system will be installed.

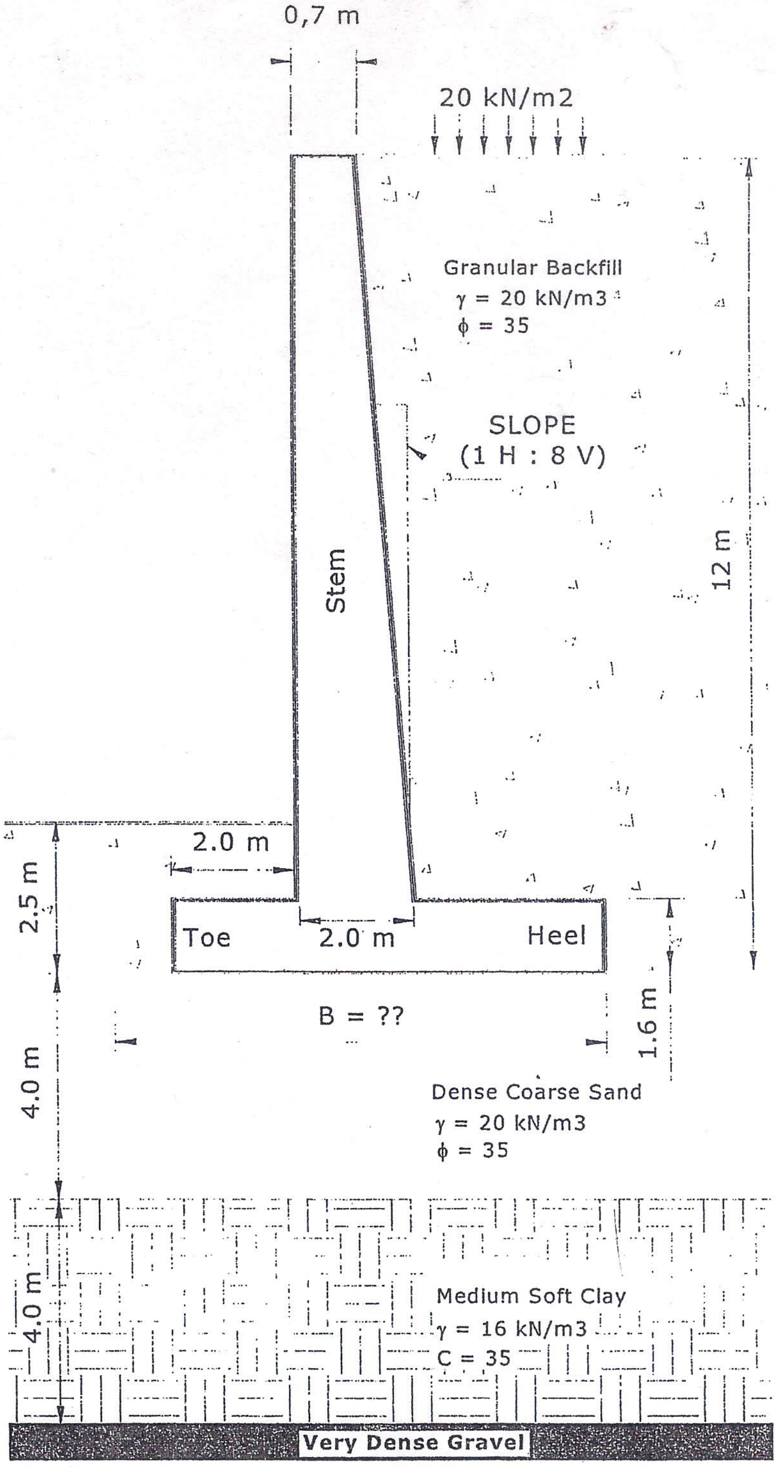
For details; see figure in the next page.

It is required to determine the following:

1. Width of the footing
2. Factor of safety against overturning
3. Factor of safety against sliding
4. Factor of safety against bearing capacity failure
5. Shear and moment in the vertical stem
6. Shear and moment in the toe projection
7. Shear and moment in the heel projection

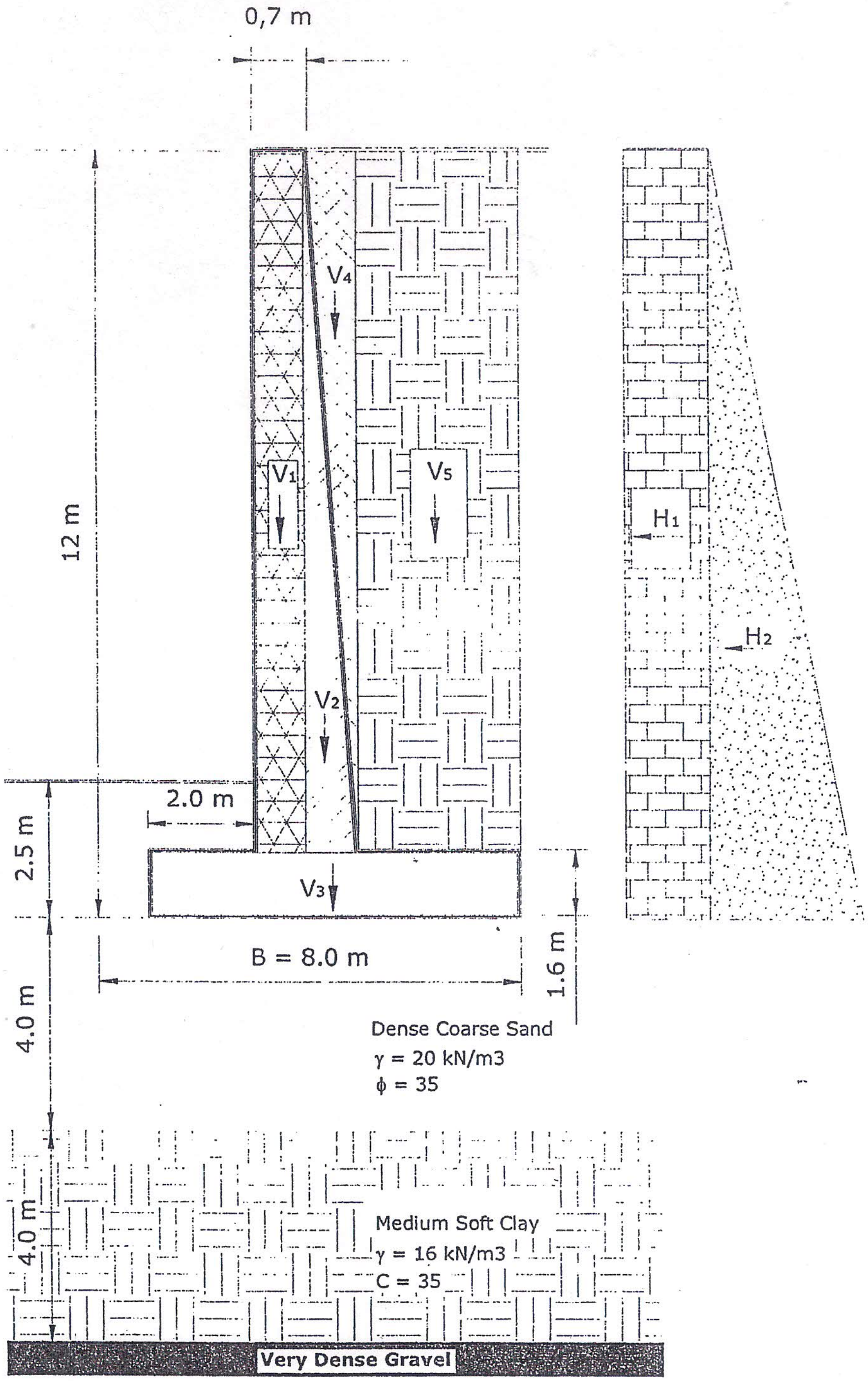
Keeping in mind that;

1. The factor of safety against overturning > 2
2. The factor of safety against sliding > **1.5**
3. γ_{con} = **24 kN/m^3**
4. Coefficient of friction between the soil and the base of the footing of the wall; that is μ is **0.5**
5. The allowable soil pressure is **280 kN/m^2**



Cantilever Retaining Wall

Solution:



Forces that the retaining wall is subjected to

$K_a = \tan^2(45 - \Phi/2)$

$\Phi = 35°$

$\Rightarrow K_a = 0.27$

Assume $B = 8.0\ m$

Pressure due to surcharge :

$P = q \times K_a = 20 \times 0.27 = 5.4\ kN/m^2$

At $Z = 0.0$ *the pressure is* 0.0

i.e. $P_a = 0.0$

At $Z = 12.0$ *the pressure is :*

$P_a = \gamma \times H \times K_a = 20 \times 12 \times 0.27 = 64.8\ kN/m^2$

Now; the following table can be prepared:

Force	kN/m (Vertical)	kN/m (Horizontal)	Arm (m)	Moment (kN.m/m)
V_1	$0.7 \times 10.4 \times 24 = 174.7$	-	2.35	+ 410.5
V_2	$10.4 \times 1.3 \times \frac{1}{2} \times 24 = 162.2$	-	3.13	+ 507.8
V_3	$1.6 \times 8.0 \times 24 = 307.2$	-	4.00	+ 1228.8
V_4	$\frac{1}{2} \times 1.3 \times 10.4 \times 20 = 135.2$	-	3.57	+ 482.7
V_5	$4 \times 10.4 \times 20 = 832$	-	6.00	+ 4992.0
H_1	-	$5.4 \times 12 = 64.8$	6.00	- 388.8
H_2	-	$\frac{1}{2} \times 64.8 \times 12 = 388.8$	4.00	- 1555.2
SUM	*Σ Vert. Forces = 1611.3*	*Σ Horiz. Forces = 453.6*		$\Sigma M_R = +7621.8$ $\Sigma M_O = -1944$

Overturning:

$$FS_{(Overturning)} = \frac{\sum M_R}{\sum M_O}$$

$$FS = \frac{7621.8}{1944} = 3.92 > 2 \Rightarrow O.K.$$

Sliding:

$$FS_{(Sliding)} = \frac{\sum F_R}{\sum F_d} = \frac{\sum V \times \mu}{\sum H}$$

$$FS = \frac{1611.3 \times (\frac{1}{2})}{453.6} = 1.78 > 1.5 \Rightarrow O.K.$$

Bearing capacity:

$$e = \frac{B}{2} - \frac{\sum M}{\sum V}$$

$$\Rightarrow e = \frac{8}{2} - \frac{7621.9 - 1944}{1611.3} = 0.48m$$

$$q_{\substack{max \\ min}} = \frac{\sum V}{A}(1 \pm \frac{6e}{B})$$

$A = 8\,m^2$ *(Note; $A = 8 \times 1$).*

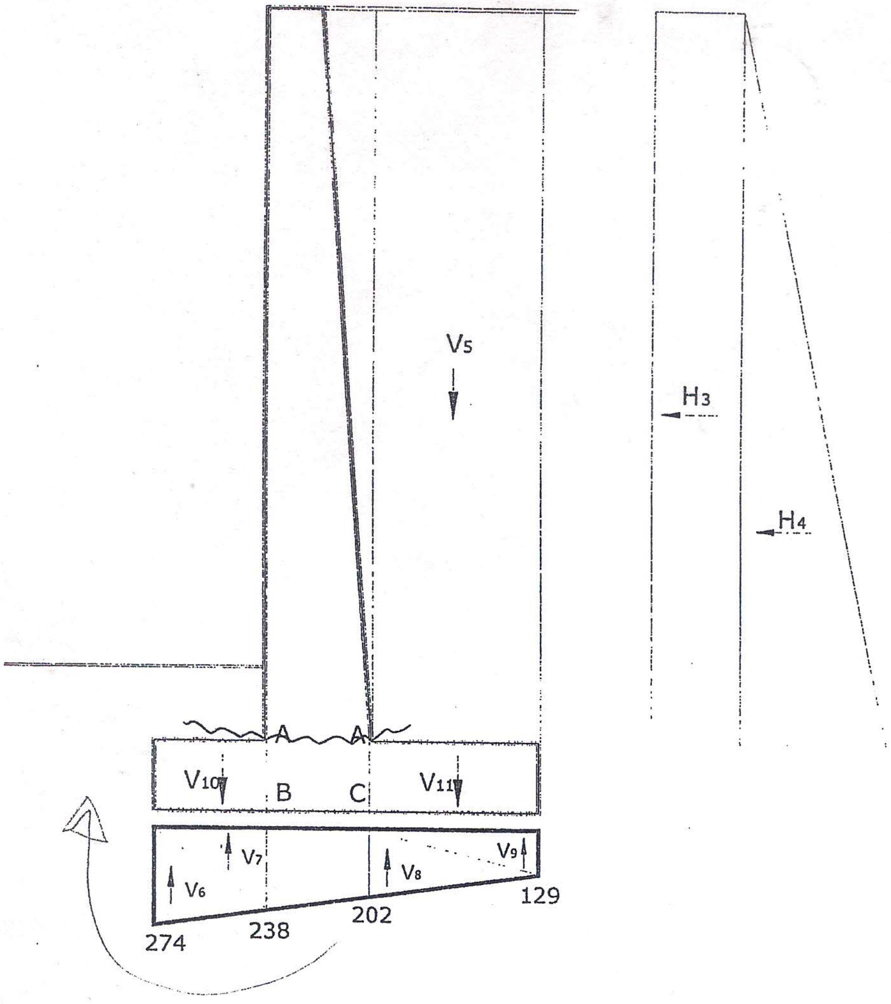
$$q_{\substack{max \\ min}} = \frac{1611.3}{8}\left(1 \pm \frac{6 \times 0.48}{8}\right)$$

$$\Rightarrow q_{max} = 273.4\,kN/m^2$$

$$\Rightarrow q_{min} = 129.5\,kN/m^2$$

Both q_{max} and q_{min} are less than the allowable soil pressure and no tension below the base of the footing

That is O.K.



Shear and Moments:

SECTION A-A			
Force	Value (kN/m)	Arm (m)	Moment (kN.m/m)
H_3	$H_3 = 0.27 \times 20 \times 10.4 = 56.2$	5.20	292.2
H_4	$H_4 = 0.27 \times \frac{1}{2} \times 20 \times (10.4)^2 = 292.0$	3.47	1012.3
Σ Shear	**348.2(LEFT)**		
Σ Moment			**1304.5 (CCW)**

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SECTION A-B			
Force	Value (kN/m)	Arm (m)	Moment (kN.m/m)
V_{10}	$V_{10} = 1.6 \times 24 \times 2.0 = -76.8$	1.00	- 76.8
V_6	$V_6 = 2 \times 274 \times \frac{1}{2} = 274$	$2 \times \frac{2}{3}$	365.3
V_7	$V_7 = 238 \times 2 \times \frac{1}{2} = 238$	$\frac{2}{3}$	158.6
Σ Shear	**435.2 (Up)**		-
Σ Moment			**447.2 (CW)**

SECTION A-C			
Force	Value (kN/m)	Arm (m)	Moment (kN.m/m)
V_5	$V_5 = -832$	2.00	- 1664
V_8	$V_8 = \frac{1}{2} \times 4 * 202 = 404$	$\frac{4}{3}$	538.7
V_9	$V_9 = \frac{1}{2} \times 4 \times 129 = 258$	$4 \times \frac{2}{3}$	688
V_{11}	$V_{11} = 4 \times 24 \times 1.6 = -135.6$	2.00	- 307.2
Σ Shear	**323.6 (Down)**		
Σ Moment			**744.5 (CW)**

Bearing Capacity of a Single Pile

Ultimate capacity of a pile; $Q_{ult} = Q_p + Q_s$

Where:

Q_p = End (point) bearing capacity of the pile at the base.

Q_s = (Skin) friction capacity of the pile perimeter.

1. Cohesive soil ($\Phi_u = 0$):

❖ **(Skin) Friction Capacity (kN)**

$Q_s = q_s A_s = \alpha C_u A_s$

Where:

q_s (or f_s) = unit skin friction (or adhesion in this case) between pile and clay (kN/m^2).

α = Adhesion factor. (See Fig. 1)

C_u = un-drained shear strength.

A_s = Shaft (friction) area.

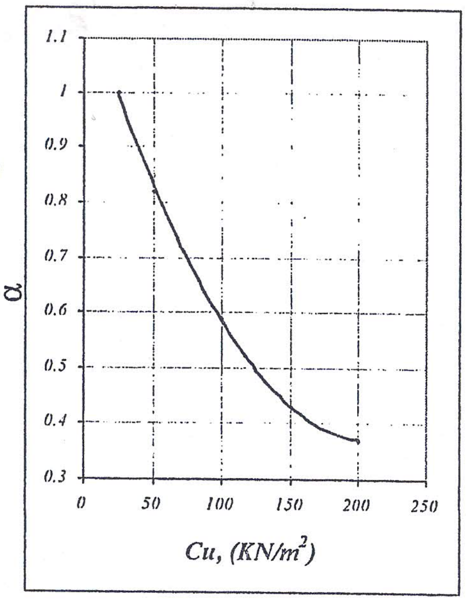


Figure (1)

If there are n different layers;

$$Q_s = \sum_{i=1}^{n} \alpha_i \times C_{ui} \times A_{si}$$

Where;

$A_s = p \times \Delta L$

P is the perimeter of the pile, then;

$$Q_s = p \times \sum_{i=1}^{n} \alpha_i \times C_{ui} \times \Delta L_i$$

❖ **End Bearing Capacity (kN)**

The general expression used to estimate the end bearing capacity of the pile is:

$Q_p = A_p\left[cN_c + \frac{1}{2}\gamma B N_\gamma + \gamma D_f N_q\right]$

Where;

N_c, N_γ, N_q bearing capacity factors for deep foundations

$Q_p = q_p A_p = N_c \times C_u \times A_p$

$N_c = 9$ is commonly used, then;

$Q_p = 9 \times C_u \times A_p$

Where;

C_u is the un-drained shear strength at pile point level.

N_c is the bearing capacity factor for cohesion for deep foundations.

A_p is the base (point) area of pile.

2. Cohesionless Soils ($C_u = 0$):

❖ **(Skin) Friction Capacity (kN)**

$$Q_s = q_s A_s = q_f A_s = K_s \times \sigma_{vo}' \times \tan\delta \times A_s$$

Where;

q_s or q_f is the unit skin friction (kPa), ($q_s = \sigma_h' \times \tan\delta$)

K_s is the coefficient of earth pressure depending on relative, volume, displacement of pile, material and shape of pile (See Table 1)

δ is angle of friction between pile and soil

Note;

The recommended value for δ for concrete piles is 20^o

σ_{vo}' is effective vertical pressure at the level considered.

$\sigma h'$ is horizontal pressure at the level considered.

$\sigma_v' = K_s \times \sigma_{vo}'$

A_s is shaft area.

If there are different cohesion-less layers:

20°

0.6(30) =

kilo pascal

kN/m2

$$Q_s = \sum_{i=1}^{n} K_{si} \times \sigma_{vo}' \times \tan\delta \times A_s$$

Since $A_s = p \times \Delta L$

$$Q_s = p \tan\delta \sum_{i=1}^{n} K_{si} \sigma_{vo}' \Delta L_i$$

Note that:

σ'_{vo} is average for each layer considered (i.e. taken as mid height)

K_s is usually constant, but if Dr's of the layers are significantly different, use Table 1

Table (1): Values of K_s

Driven Pile (Displacement Pile)		Small Displacement Pile		Bored Pile
Low Dr	**High Dr**	**Low Dr**	**High Dr**	
1.0	2.0	0.5	1.0	0.5

❖ **End Bearing Capacity (kN)**

(Note that the term with N_γ in the general equation is comparatively small, so it is cancelled)

$$Q_P = q_P A_P = N_q \times \sigma_{vo}' \times A_P$$

Where;

q_p is the unit base resistance (kPa).

A_p is the base area of pile.

N_q is the bearing capacity factor for deep foundations, see Table 2.

σ'_{vo} is the vertical effective stress at the base level of the pile.

Note;

The vertical effective stress is considered to be constant after 20D or 20B which is called the critical depth

Table (2): Values of N_q

Φ'	20	25	28	30	32	34	36	38	40	42	45
N_q Driven	8	12	20	25	35	45	60	80	120	160	230
N_q Bored	4	5	8	12	17	22	30	40	60	80	115

Problem # (16)

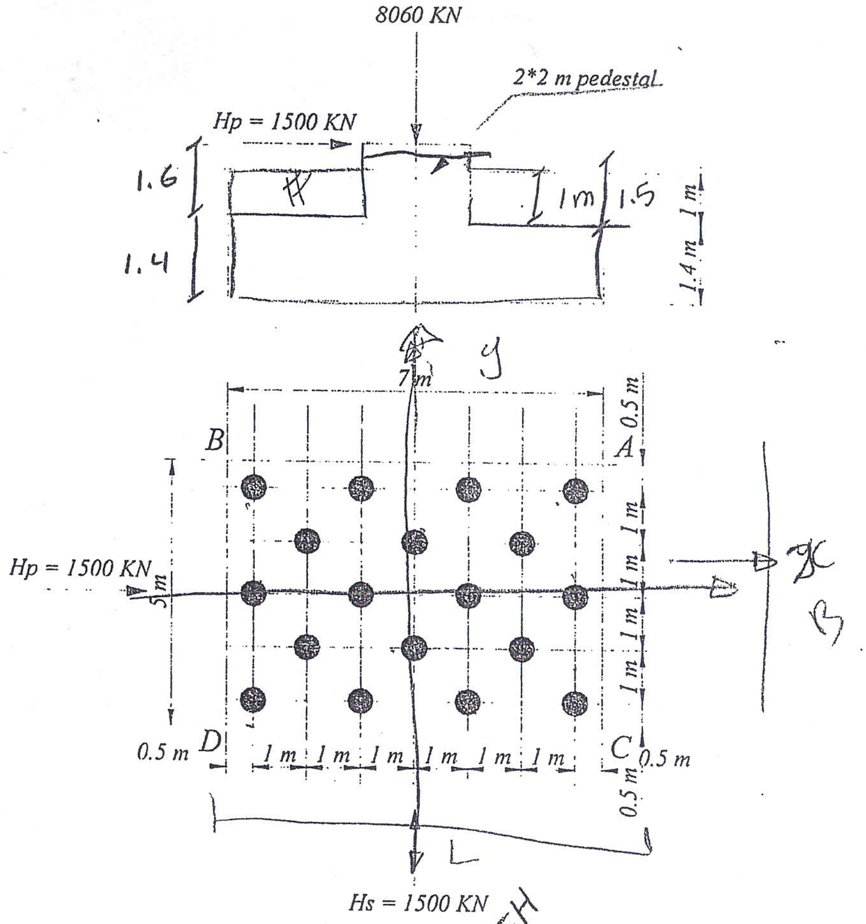
A rectangular footing is acted by a vertical load of 8060 kN and two horizontal forces of 1500 kN in both the long and the short directions with a distance of 0.6 m from the ground surface. Determine:

1. The base pressure at the four corners if the footing was on soil.
2. Pile loads at the four corners if the footing was on piles whose pattern as shown.

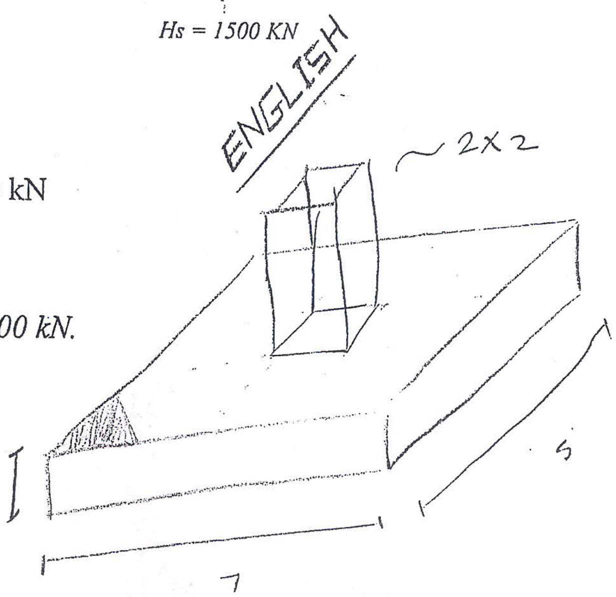
Solution:

$\gamma_{soil} = 20\ kN/m^3$

$\gamma_{conc} = 24\ kN/m^3$



Pedestal	$2\times2\times1.5\times24 = 144$ kN
Footing	$5\times7\times1.4\times24 = 1176$ kN
Soil	$(5\times7-2\times2)\times1.0\times20 = 620$ kN
Column load	8060 kN



Total load = 144+1176+620+8060 = 10,000 kN.

1. *Base Pressure at the Four Corners:*

$$q_{\substack{max \\ min}} = \frac{\sum Q}{A}\left(1 \pm \frac{6e_L}{L} \pm \frac{6e_B}{B}\right)$$

$$e_L = e_B = \frac{1500 \times 3}{10{,}000} = 0.45m$$

$$q_{\substack{max \\ min}} = \frac{10{,}000}{5 \times 7}\left(1 \pm \frac{6 \times 0.45}{7} \pm \frac{6 \times 0.45}{5}\right)$$

$$Q_{\substack{max \\ min}} = 285.7 \pm 111.4 \pm 154.3$$

$q_A = 285.7 + 111.4 + 154.3 = 551$ kN/m^2

$q_B = 285.7 - 111.4 + 154.3 = 329$ kN/m^2

$q_C = 285.7 + 111.4 - 154.3 = 243$ kN/m^2

$q_D = 285.7 - 111.4 - 154.3 = 20$ kN/m^2

2. *Pile Loads at the Four Corners.*

$$Q = \frac{\sum Q}{n} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x}$$

$$e_L = e_B = 0.45m$$

$$e\sum Q = M_L = M_B = 4500 KN/m$$

$I_B = (1^2 + 3^2) \times 3 \times 2 + 2^2 \times 2 \times 2 = 76$ pile.m^2

$I_L = 2^2 \times 4 \times 2 + 1^2 \times 2 \times 3 = 38$ pile.m^2

$$Q = \frac{10{,}000}{18} \pm \frac{4500}{76} \times 3 \pm \frac{4500}{38} \times 2$$

$$Q_{\substack{max \\ min}} = 556 \pm 178 \pm 237$$

$Q_A = 556 + 178 + 237 = 971$ KN/m^2

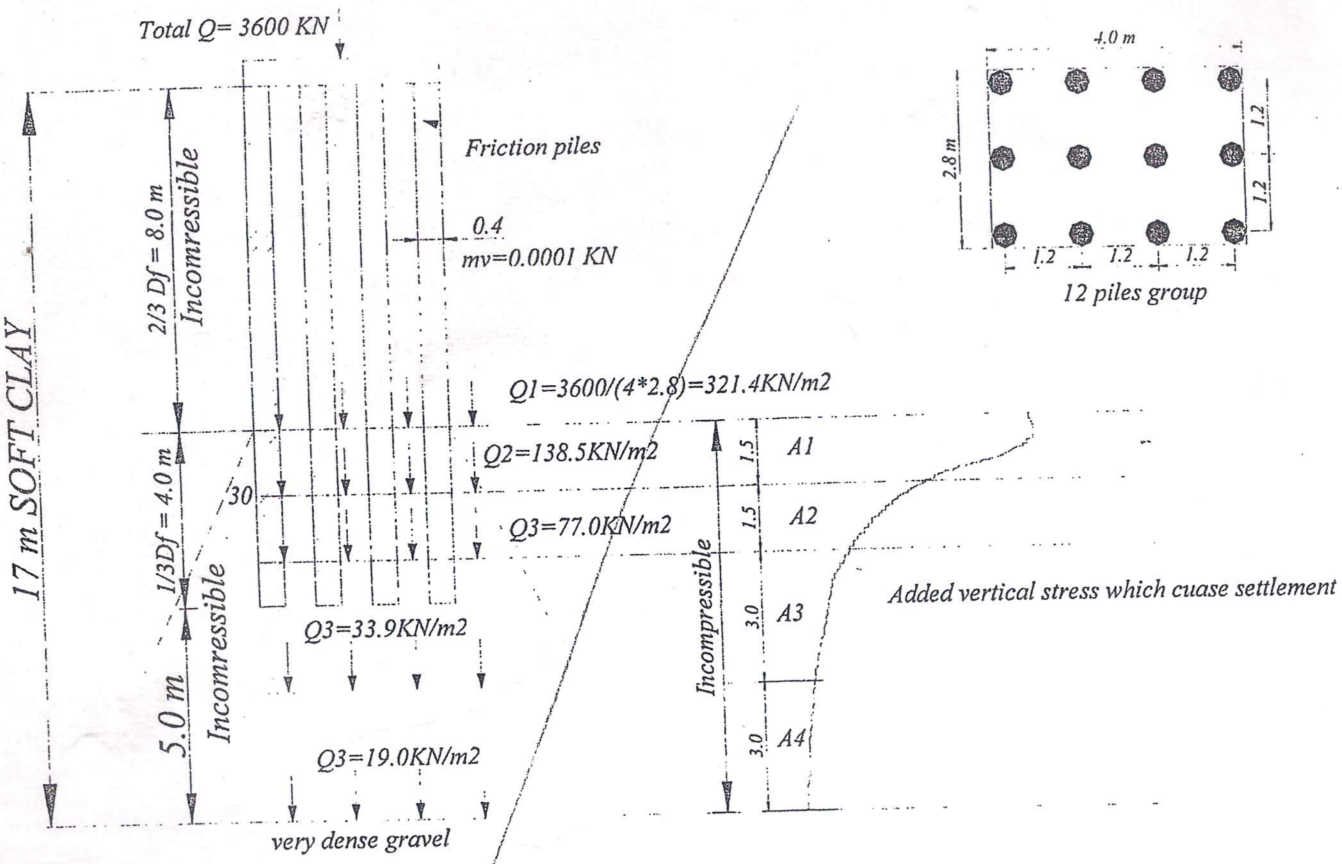
$Q_B = 556 - 178 + 237 = 615$ KN/m^2

$Q_C = 556 + 178 - 237 = 497$ KN/m^2

$Q_D = 556 - 178 - 237 = 141$ KN/m^2

Problem # (17)

For the group of friction piles shown; determine the settlement of the clay layers.



Solution:

$S = \Delta P \times \Delta H \times m_v = A_1 \times m_v$

A = Pressure area

S is the settlement of the clay layer

$A_1 = 0.5 \times (321.4 + 138.5) \times 1.5 = 345$ kN/m.

$A_2 = 0.5 \times (138.5 + 77.0) \times 1.5 = 162$ kN/m.

$A_3 = 0.5 \times (77.0 + 33.9) \times 3.0 = 166$ kN/m.

$A_4 = 0.5 \times (33.9 + 19) \times 3.0 = 79$ kN/m.

$S_1 = 345 \times 0.0001 = 0.0345$ m

$S_2 = 162 \times 0.0001 = 0.0162$ m

$S_3 = 166 \times 0.0001 = 0.0166$ m

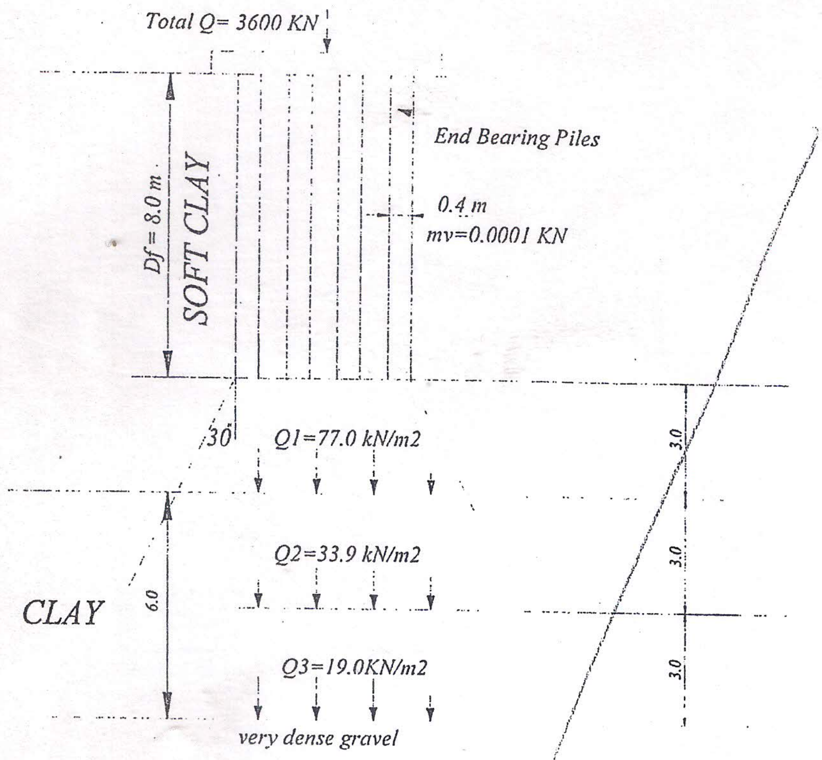
$S_4 = 79 \times 0.0001 = 0.0079$ m

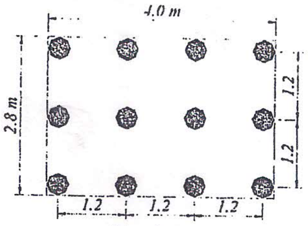
Total Settlement = 0.0752 m = 75.2 mm

Note that almost the half of the settlement of the pile group is due to compression of the upper 1.50 thick layer.

Problem # (18)

Determine the settlement of the group of point bearing piles shown.





12 piles group

Solution:

$S = \Delta P \times \Delta H \times m_v = A_L \times m_v$

A = Pressure area

S is the settlement of the clay layer

Note; Q is used instead of ΔP in the following equations

$$Q_1 = \frac{3600}{(4+2\times 3\times \tan 30)\times(2.8+2\times 3\times \tan 30)} = 77.0\ kN/m^2$$

$$Q_2 = \frac{3600}{(4+2\times 6\times \tan 30)\times(2.8+2\times 6\times \tan 30)} = 33.9\ kN/m^2$$

$$Q_3 = \frac{3600}{(4+2\times 9\times \tan 30)\times(2.8+2\times 9\times \tan 30)} = 19\ kN/m^2$$

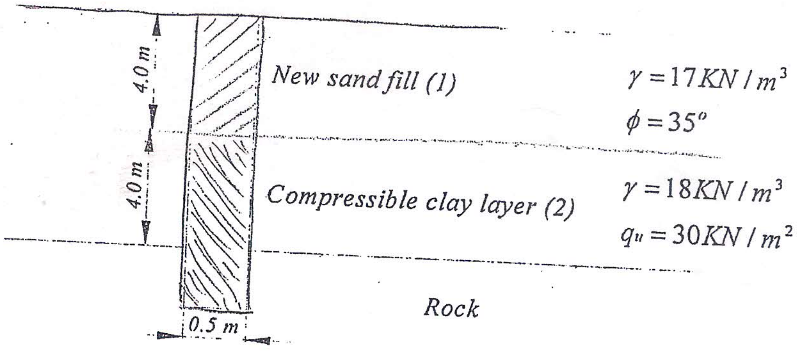
$$S_1 = \frac{1}{2}\times(77.0+33.9)\times 3.0\times 0.0001 = 0.0166\ m$$

$$S_2 = \frac{1}{2}\times(33.9+19.0)\times 3.0\times 0.0001 = 0.0079\ m$$

Total Settlement $S = 0.0166 + 0.0079 = 0.0245\ \text{m} = 24.5\ \text{mm}$

Problem # (19)

A reinforced concrete circular pile is driven to the soil as shown below. Calculate the negative skin friction force acting on the pile.

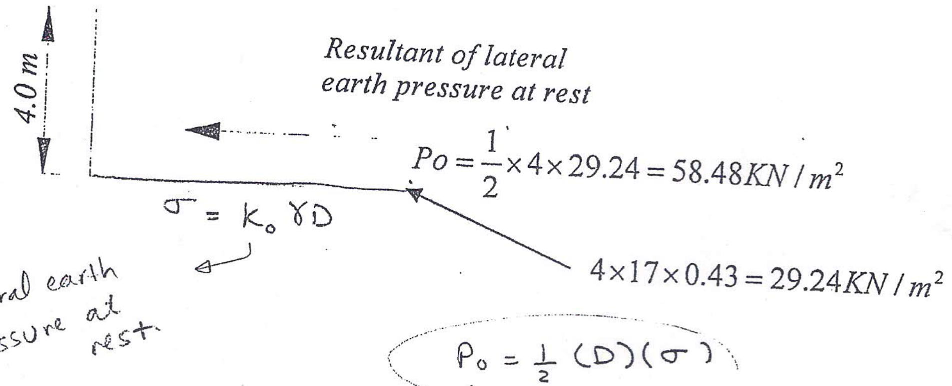


Solution:

$K_o = 1 - \sin\phi = 0.43$

$\delta = \frac{2}{3}\tan\phi = 0.467$

$C_u = \frac{q_u}{2} = 15 KN/m^2$



$F_1 = P_o \delta \pi D$

$F_1 = 58.48 \times 0.467 \times \pi \times 0.5 = 43 KN$

$F_2 = \pi D h \alpha_{Cu}$

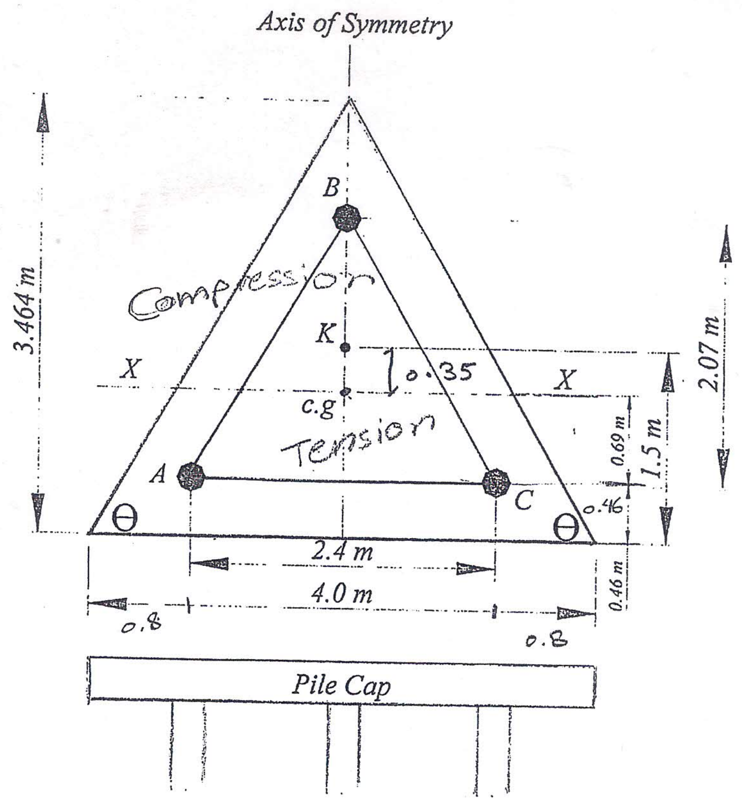
$F_2 = \pi \times 0.5 \times 4 \times 15 = 94.3 KN$

Total negative skin friction, $F = F_1 + F_2 = 137.3$ kN/pile

Problem # (20)

A pile supported foundation is given below. A vertical force of 1200 kN (including all the loads on foundation) is acting downward at point K.

Determine the loads carried by piles A, B and C.



Solution:

$$\tan\theta = \frac{3.464}{2} \Rightarrow \theta = 60''$$

$$\tan\frac{\theta}{2} = \tan 30^0 = \frac{x}{0.8} \Rightarrow x = 0.46m.$$

$\therefore$ *Eccentricity*:

$$e = 1.5 - (0.69 + 0.46) = 0.35m$$

$\therefore$ *Moment*:

$$M = 1200 \times 0.35 = 420 KN.m$$

Moment of inertia of piles:

$$I_{x-x} = 1 \times 1.38^2 + 2 \times 0.69^2 = 2.86 Pile.m^2$$

Then; the force on piles A, B, and C will be;

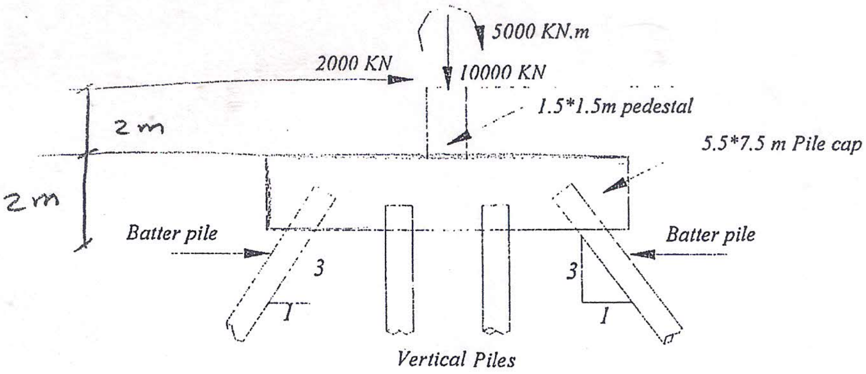
$$Q_A = Q_C = \frac{1200}{3} - \frac{420}{2.86} \times 0.69 = 299 kN$$

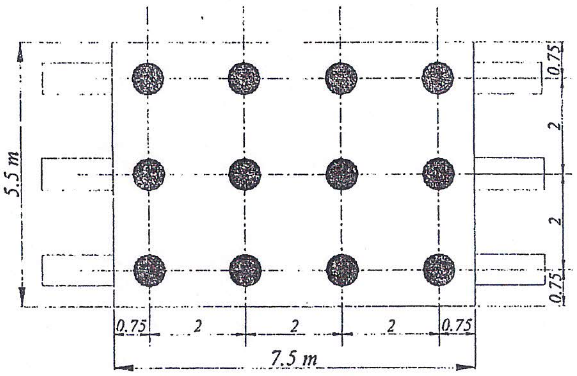
$$Q_B = \frac{1200}{3} + \frac{4200}{2.86} \times 1.38 = 603 kN$$

Problem # (21)

One of the exterior columns of a steel frame structure is acted by a vertical force of 10,000 kN, horizontal force of 2000 kN and a moment of 5000 kN.m. Due to a poor foundation soil; it was decided to support that column by a pile foundation consisting of a twelve pile group. Piles are concrete filled steel pipe piles. Allowable axial compressive load on each pile is 2000 kN. It is also predicted that each pile can also resist a horizontal force of 60 kN. Passive resistance of soil will not be considered.

Is the selected pile group satisfactory?





Solution:

Column load = 10,000 kN

Weight of pedestal = 1.5 ×1.5 ×2 ×24 = 108 kN

Weight of pile cap = 5.5 ×7.5 ×2 ×24 = 1,980 kN

Weight of soil = (5.5 ×7.5 – 1.5 ×1.5) ×2 ×20 = 1,560 kN

Total vertical loads = 10,000 + 108 + 1,980 + 1,560 = 13,648 kN

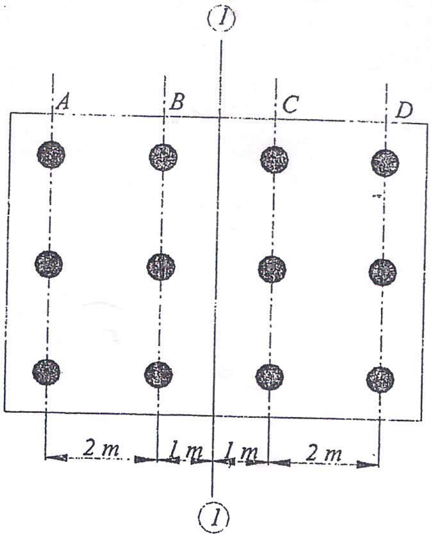
Moment = 5000 + 4 ×2000 = 13000 kN.m

Horizontal loads = 2000 kN

$$I_{1-1} = 2\times3\times(1^2+3^2) = 60\, pile.m^2$$

$$Q_n = \frac{\sum Q}{n} \pm \frac{\sum M}{I_{1-1}} \times X_n$$

$$Q_n = \frac{13648}{12} \pm \frac{13000}{60} \times X_n = 1137 \pm 217 X_n$$



Then;

$Q_A = 1137 - 217\times3 = 486\ kN$

$Q_B = 1137 - 217\times1 = 920\ kN$

$Q_C = 1137 + 217\times1 = 1354\ kN$

$Q_D = 1137 + 217\times3 = 1788\ kN$

Total load $= 4548\times3 = 13644 \approx 13648$

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Unbalanced horizontal force $= 2000 + 3\times\frac{486}{3} - 3\times\frac{1788}{3} = 698 KN$

Now; $\frac{698}{12} = 58 kN/pile \prec 60 \Rightarrow O.K.$

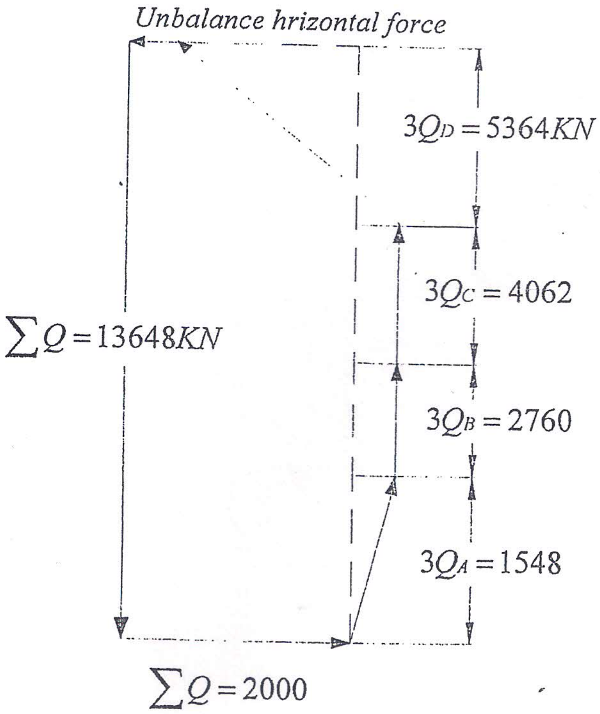
Maximum axial load:

$$R_D = \frac{Q_D}{\cos\alpha}$$

$$\tan\alpha = \frac{1}{3}$$

$$\cos\alpha = 0.95$$

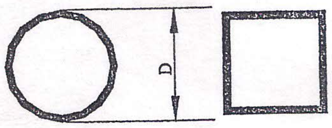
$$RD = \frac{1.788}{0.95} = 1884 kN \prec 2000 kN \Rightarrow O.K.$$



Deep Foundations

1. Equations:

Q_u = Ultimate Pile Capacity.
Q_P = Load-Capacity of pile point.
Q_s = Skin friction resistance.
L = Pile length.
q' = Overburden pressure at pile tip. → effective stress at pile tip
D = Pile dimension.
A_P = Area of the pile cross-section.



Pile cross-section

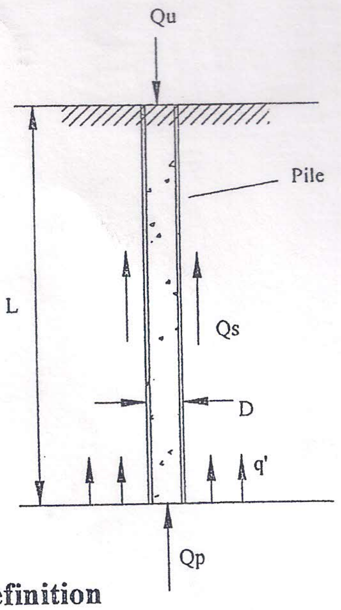


Figure 1, Parameters' definition

2. Load – Capacity of Pile Point:

$Q_P = A_P\, q'\, N_q^*$

Or, $Q_P = A_P\, q_l$

Whichever is smaller.

Where;

q_l (kPa) = $50\, N_q^* \tan\Phi$
N_q^* = Bearing capacity factor for deep foundations.

$q_p = c N_c^* + q N_q^* + \gamma B N_\gamma^*$

Q_p (load capacity of pile point)
$Q_p = A_p q_p$

clay ($\phi = 0$)
$Q_p = c N_c^* A_p = q_{cu} A_p$
cohesion

sand ($c = 0$)
$Q_p = q' N_q^* A_p \leq A_p q_l$
where $q_l = 50 N_q^* \tan\phi$
figure 11.12

3. Skin Friction Resistance:

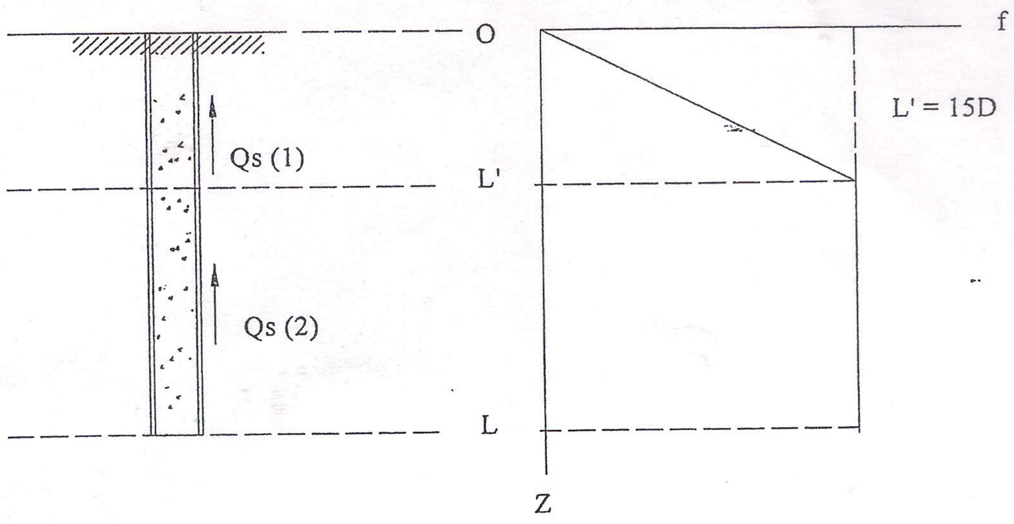


Figure 2 . Variation of Skin Friction with Depth

$Q_s\,(1) = P\,L'\,f_{av}$

Q_s = Σ p ΔL f unit frictional resistance.

$Q_s\,(2) = p\,(L - L')\,f$

$Q_s = Q_s\,(1) + Q_s\,(2)$

Where;

$f = K\,\sigma v'\,\tan\sigma$ (skin friction)

σ_v' = overburden pressure

K = lateral earth pressure = 0.5 + 0.008Dr (Dr = Relative Density in Percent)

δ = friction angle between soil and pile (usually 0.6 Φ)

P = perimeter of the pile

f_{av} = average skin friction (From 0 to L it is f / 2)

Friction pile ... perimeter of pile.

Q_s = Σ P ΔL f

Clay: α method, f = α cu

Sand: f = ..., Table, δ = ...

4. Efficiency of Pile Group:

Efficiency, $\eta = \dfrac{2(n_1 + n_2 - 2) \times d + 4D}{p n_1 n_2}$

Where the parameters are as defined in Figure 3.

Ultimate load capacity of pile group:

$Q_u(g) = \sum Q_u * \eta$

If $\eta \geq 1.0$; use $\eta = 1.0$

Allowable load capacity of pile group:

$Q_a(g) = Q_u(g) / SF$

Where; $2.5 \leq SF \leq 4.5$

$F.S = \dfrac{Q_{u(g)}}{Q_{a(g)}}$

2.5 – 4.5

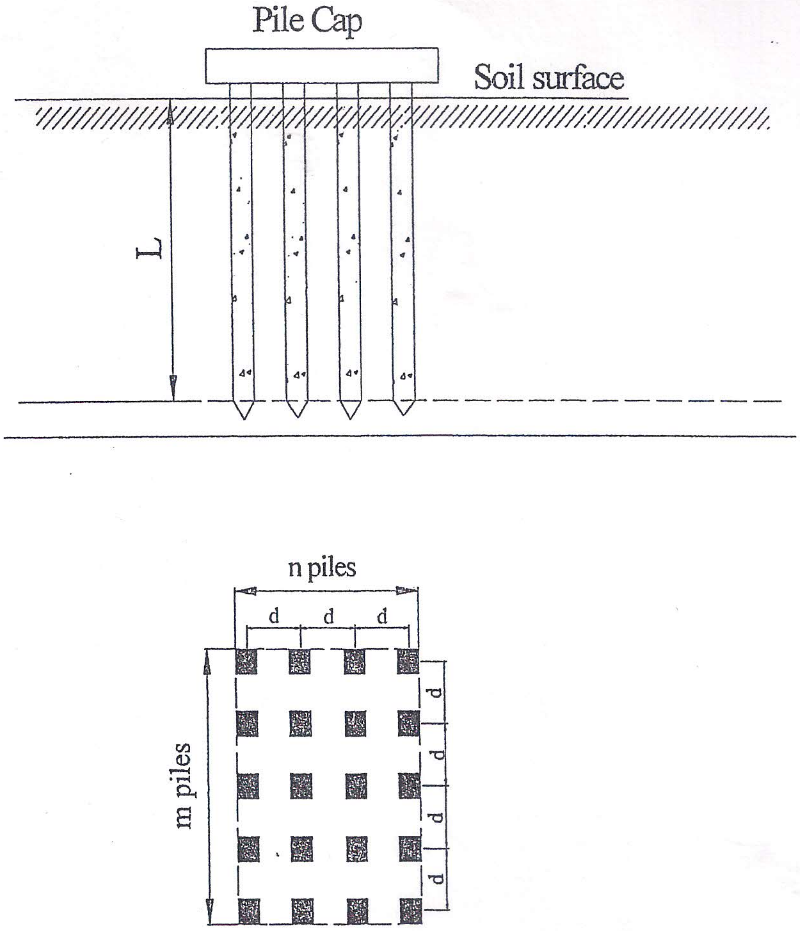
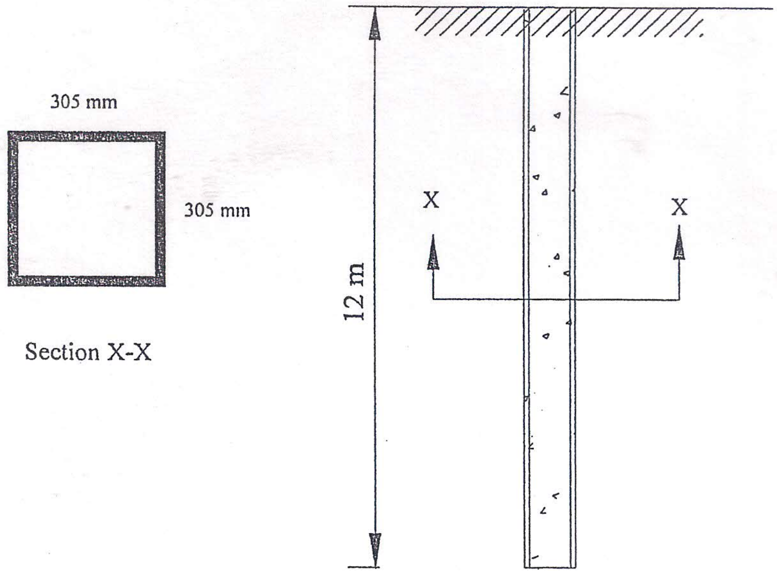


Figure 3. Pile group

Problem # (22)

Consider a precast concrete pile 12 – m long in a homogenous sandy soil layer. The pile cross section is 305 mm × 305 mm, the dry unit weight of sand is, $\gamma_d = 16.8\ kN/m^3$, and the soil friction angle $\Phi = 35^o$.

Calculate the ultimate point load that the pile can carry.



Solution:

For $\Phi = 35^o$, $Nq^* = 120$. Thus;

$q' = \gamma_d L = 16.8 \times 12 = 201.6$ kPa

$Q_p = A_p q' N_q^* = (0.305)^2 \times 201.6 \times 120 = 2250.46$ kN

OR

$q_1 = 50 N_q^* \tan \Phi = 50 \times 120 \times \tan 35 = 4201.25$

$Q_p = A_p q_1 = (0.305)^2 \times 4201.25 = 390.82$ kN

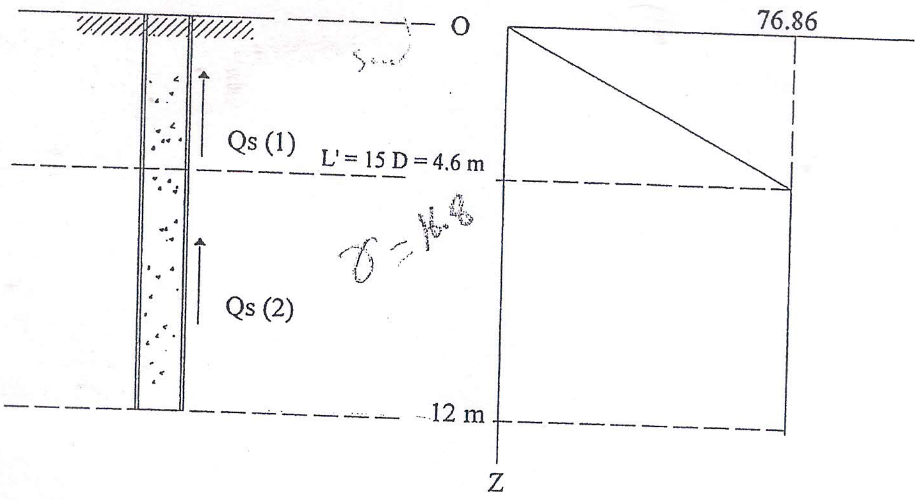
Thus the point ultimate load of the pile that is Q_p is:
$Q_p = 390.82$ kN

Problem # (23):

Consider a pre-cast concrete pile in Problem # (21):

1. Estimate the ultimate skin frictional resistance Q_s
2. Calculate the total ultimate load of the pile, Q

Assume that for the given dry unit weight the relative density of the sand Dr = 100 %



Solution:

The unit skin friction is estimated as: $f = K\,\sigma_v'\tan\delta$

Where: σ_v' = overburden pressure = γz (See Figure Above)

For depth z from 0 to 15 D : $\sigma_v' = 16.8\,z$ (kPa)

For depth $z \geq 15$ D : $\sigma_v' = 16.8\,(15 * 0.305) = 76.86$ kPa (Constant)

K = lateral earth pressure = 0.5 + 0.008 Dr = 0.5 + (0.008 * 100) = 1.3

$$Q_s(1) = PL' f_{av} = (4\times0.305\times4.6)\left[\frac{1.3\times76.86\times\tan(0.6\times35)}{2}\right] = 107.64\ kN$$

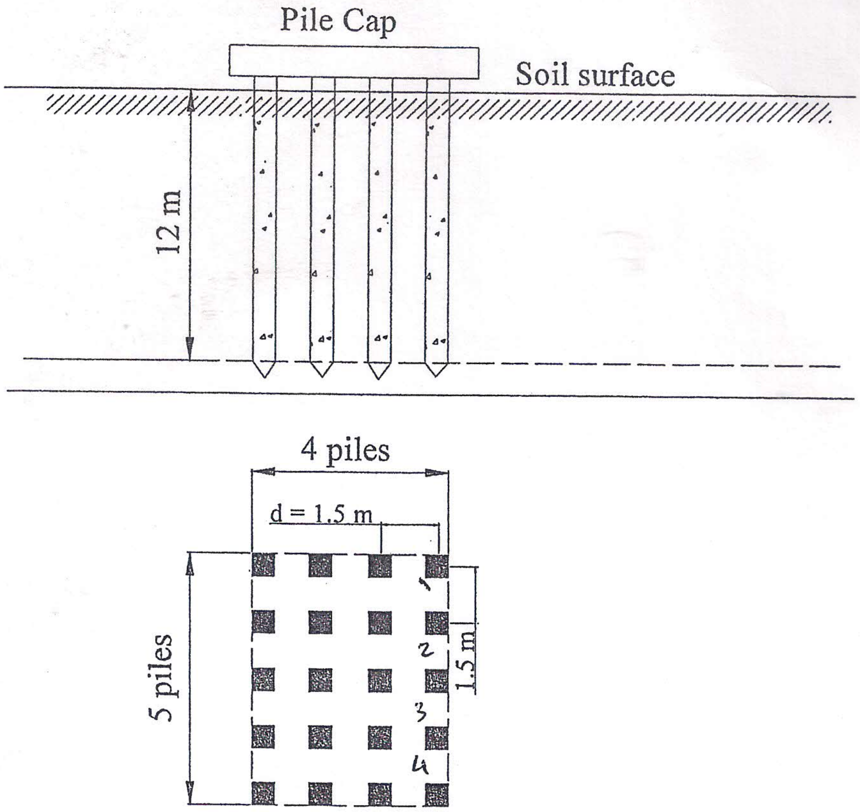
$$Q_s(2) = P\,(L-L')\,f = [\,4\times0.305\,]\,[\,12-4.6\,]\,[1.3\times76.86\times\tan(0.6\times35)] = 346.27\ kN$$

$Q_s = Q_s(1) + Q_s(2) = 107.64 + 346.27 = 453.91$ kN

Total load = Q_{pile} = (problem # 21) + Q_s = 390.82 + 453.91 = 844.73 kN

Problem # (24):

Estimate the total load that the pile group shown below can carry. Note that; every single pile is identical to that in problems # (21) and # (22).



Solution:

Total load that the single pile can carry Q = 844.73 kN (Problem # (21) and # (22))

m = 5

n = 4

so, m × n = 20 piles

Q_T = 844.73 × 20 = 16894.6 kN

The efficiency of the group:

$$\eta = \frac{2(m+n-2)\times d + 4D}{pmn}$$

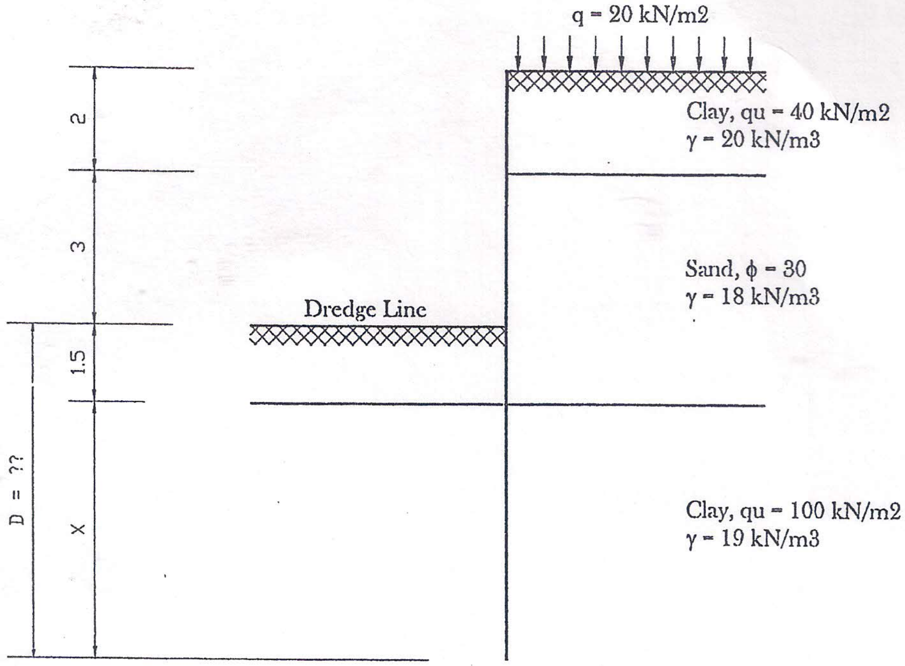
$$\eta = \frac{2(5+4-2)\times(1.5)+(4\times 0.305)}{(4\times 0.305)\times 5\times 4} = 0.91 \text{ (or 91 \%)}$$

Therefore; the total load that can be carried by the pile group is;

Q_{TG} = 16894.6 × 0.91 = 15374.09 kN.

Problem # (25)

Determine the depth of penetration and the maximum bending moment of a cantilever sheet pile wall constructed in a three layered soil system as shown below. What should be the required section modulus of a steel sheet pile if the allowable stress of steel is 150 MPa?



Solution:

$\sigma_a = (q + \sigma_v') \times K_a - 2 \times C\sqrt{K_a}$

$\sigma_p = (q + \sigma_v') \times K_a + 2 \times C\sqrt{K_a}$

For the clay layer; $\Phi = 0.0°$, that means $K_a = K_p = 1.0$

For the sand layer; $\Phi = 30.0°$, that means $K_a = \frac{1}{3}$ and $K_p = 3.0$

From soil mechanics, q = 2C

Pressure Diagram:

- At depth (Z) = 0.0 m

$\sigma_a = (20 + 0.0) \times 1 - 2 \times 20\sqrt{1} = -20 \text{ kN/m}^2$

- At depth (Z) = 2.0 m (Clay Layer)

$\sigma_a = (20 + 2 \times 20) \times 1 - 2 \times 20\sqrt{1} = 20 \text{ kN/m}^2$

- At depth (Z) = 2.0 m (Just below the Clay Layer)

$\sigma_a = (20 + 2 \times 20) \times \frac{1}{3} - 2 \times 0.0 \times \sqrt{\frac{1}{3}} = 20 \text{ kN/m}^2$

English

- At depth (Z) = 5.0 m (Dredge Line)

$$\sigma_a = (20+2\times20+3\times18)\times\frac{1}{3}-2\times0.0\sqrt{\frac{1}{3}} = 38 \text{ kN/m}^2$$

- At depth (Z) = 6.5 m

$$\sigma_a = (20+2\times20+3\times18+1.5\times18)\times\frac{1}{3}-2\times0.0\sqrt{\frac{1}{3}}$$

$$\sigma_p = (0.0+1.5\times18)\times3+0.0$$

$$\sigma_p-\sigma_a = 34.0\, kN/m^2$$

If we assume that $\sigma_p-\sigma_a = 0.0$ and solve for the depth;

It will be found out that this will occur at a depth of 0.79 m below the dredge line.

- At depth (Z) = 6.5 m to (6.5 + X) m

$$\sigma_a = (20+2\times20+3\times18+1.5\times18+19\times X)\times1-2\times50\times\sqrt{1}$$

$$\sigma_p = (0.0+1.5\times18+19\times X)\times1+2\times50\times\sqrt{1}$$

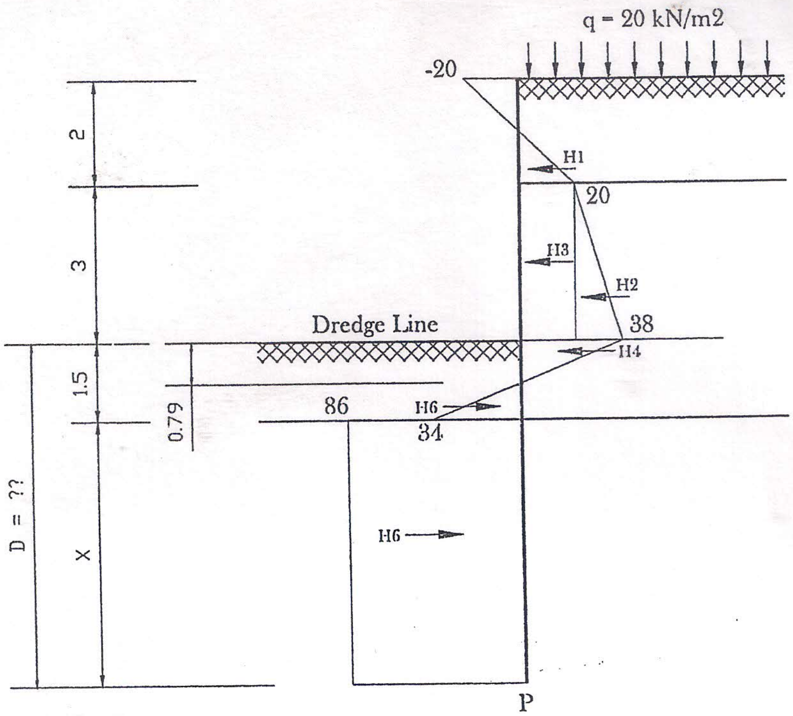
Now; the net pressure at any depth larger than 6.5 m is:

$$\sigma_p-\sigma_a = 86.0\, kN/m^2$$

Note; the depth of penetration of the sheet pile will be 1.5 + X

Now; the following table can be arranged:

Force #	Force (kN / m)	Arm (m)	Moment (kN.m / m)
H_1	$1\times\frac{1}{2}\times20=10.0$	4.83 + X	10 X + 48.3
H_2	$20\times3=60.0$	3.00 + X	60 X + 180
H_3	$18\times3\times\frac{1}{2}=27.0$	2.50 + X	27 X + 67.5
H_4	$38\times0.79\times\frac{1}{2}=15.0$	1.24 + X	15 X + 18.6
H_5	$-34\times0.71\times\frac{1}{2}=-12.0$	0.24 + X	-12 X – 2.8
H_6	$-86\times X=-86X$	$\frac{X}{2}$	-43 X^2



Now; taking the sum of the moment about the base of the wall (point P) will lead to:

$\Sigma M_{Base} = 0.0$

$-43 \times X^2 + 100 \times X + 311.6 = 0.0 \Rightarrow$

$X = 4.1\,m$

Depth of penetration of the sheet pile will be;

$D_o = 1.5 + 4.1 = 5.6$ m

$D = 1.2 \times 5.6 = 6.7$ m

Maximum bending moment M_{max} occurs at the point of zero shear

Shear is the derivative of the moment, so, shear equation will be:

V (shear) = - 86 X + 100

V = 0.0 at a depth of 1.163 m

M_{max} (Moment at a depth of 1.163 m) = 369.7 kN.m / m

Section Modulus is calculated from the following equation:

$$S = \frac{M_{max}}{\sigma_{all}} = \frac{369.7 \times 10^9}{150 \times 10^3}$$

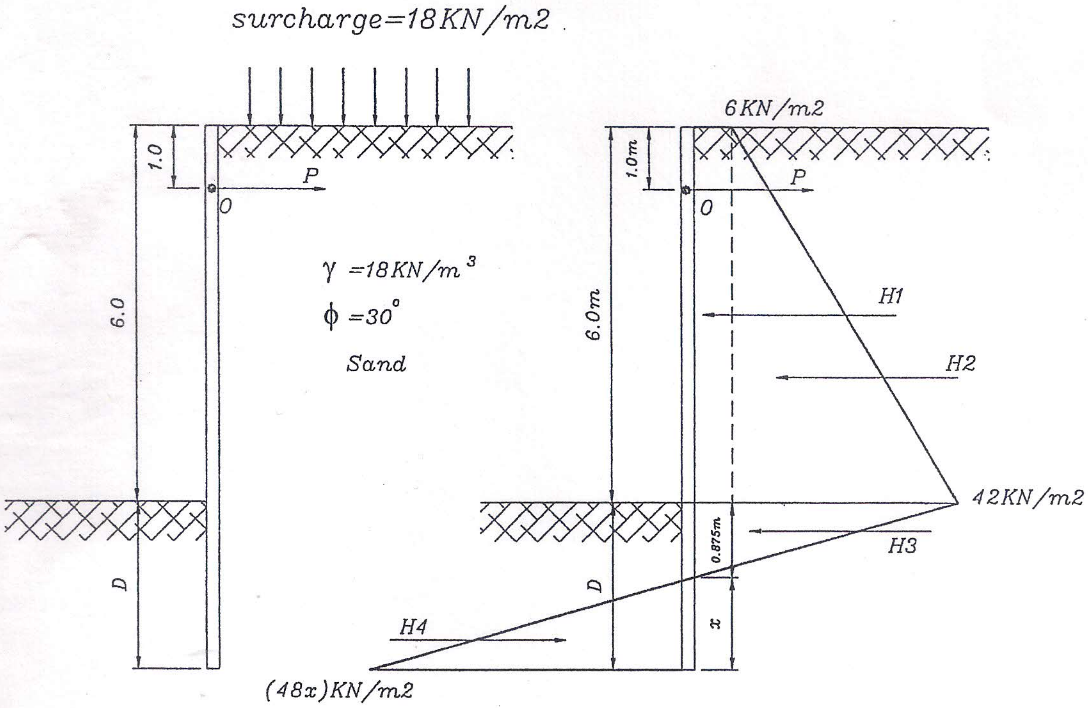
$$= 2467 \times 10^3\,mm^3$$

Problem # (26)

An anchored steel sheet pile wall in a hemogeneous sand deposit is to be designed and constructed. Determine the depth of penetration, maximum bending moment and anchored pull by:

1- Free earth support method.
2- Fixed earth support method.

Free earth support method:



Solution:

Force (KN)	Arm about anchor, point O (m)	Moment (kN.m)
H_1= 36	2	72
H_2= 108	3	324
H_3= 18.375	5.292	97.24
H_4= $-24x^2$	5.875+2/3 x	$-141x^2-16x^3$
		$-16x^3-141x^2+493.24$

Taking the moment about the anchor equal zero

$$\sum M_O = -16x^3 - 141x^2 + 493.24 = 0$$

$$x^3 + 8.813x^2 - 30.828 = 0$$

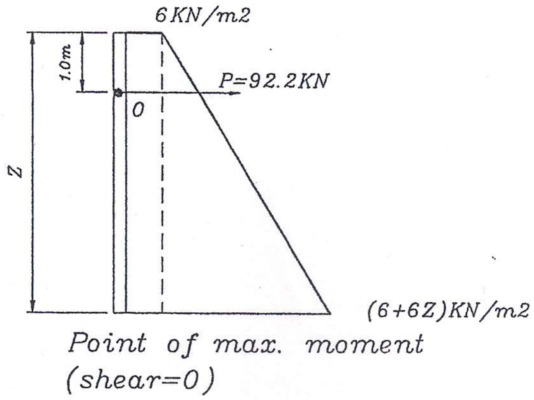
By trial & error $\Rightarrow \therefore\ x \cong 1.71m$

Penetration depth = 0.875+1.71 = 2.6 m

$$\sum F_x = 0$$

$$\therefore 36 + 108^2 + 18.375 - 24 \times 1.71^2 - P = 0$$

$\therefore P = 92.2kN$ **Per meter width of the wall**



Zero shear point:

H_1+H_2 = 36+108 = 144kN >P

Zero shear point must be at the depth from (1-6) m

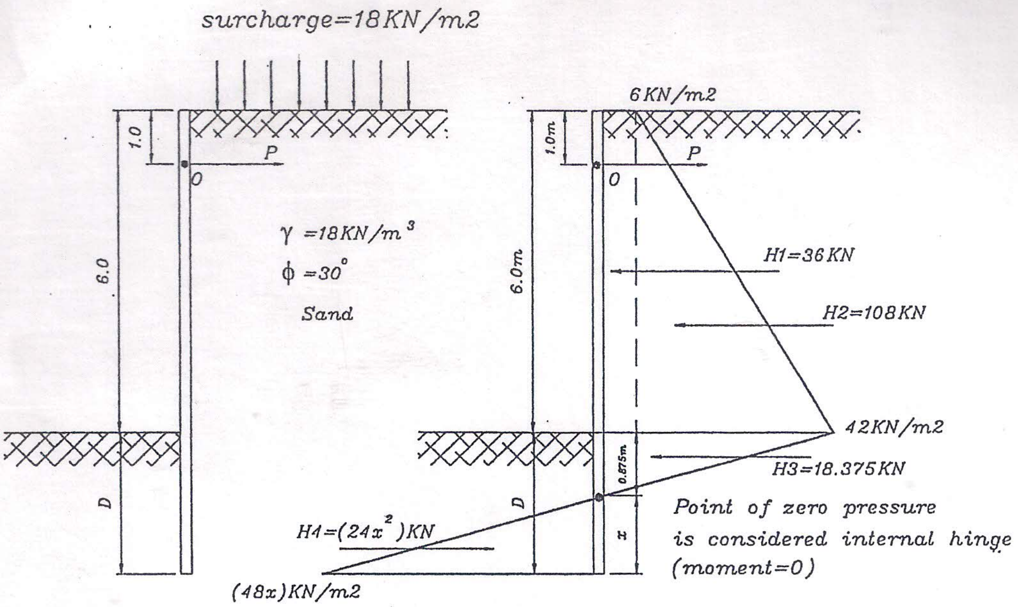
$$\sum F_x = 0$$

$$\therefore 6Z + \frac{1}{2} \times 6Z \times Z - 92.2 = 0$$

$$3Z^2 + 6Z - 92.2 = 0 \Rightarrow Z = 4.63\,m$$

$$M_{max} = 92.2 \times (4.63 - 1) - 6 \times \frac{4.63^2}{2} - \frac{1}{2} \times 27.78 \times \frac{4.63^2}{3} = 171.12\,kN.m$$

Fixed earth support method:-

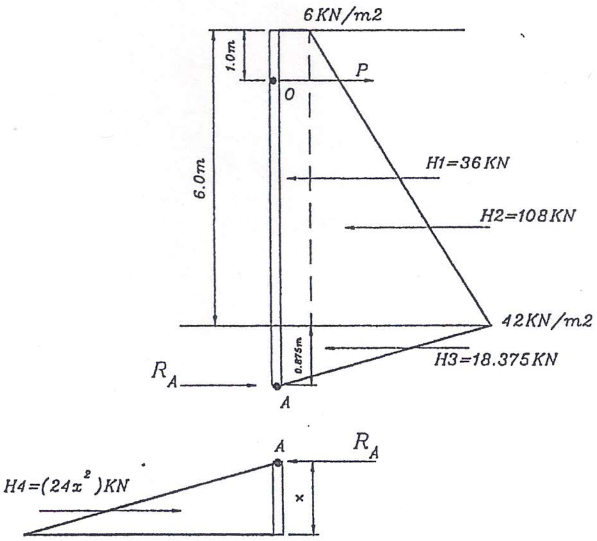


Solution :

$$\sum M_A = 0$$

$$\therefore 5.875 \times P - 36 \times 3.875 - 108 \times 2.875 - 18.375 \times 0.583 = 0$$

P = 78.42 kN **per meter width of the wall**



$\sum F_x = 0$

$\therefore 78.42 - 36 - 108 - 18.375 + R_A = 0$

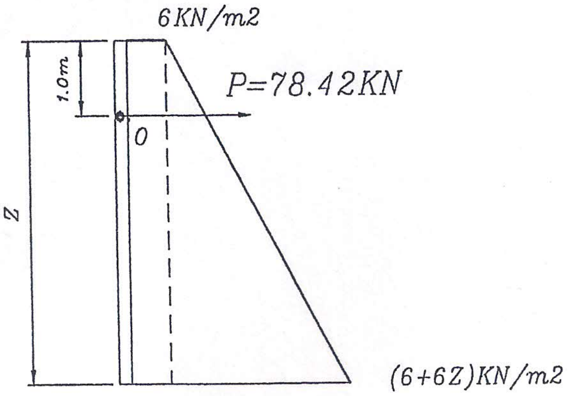
$\therefore R_A = 84.375 kN$

$\sum M_O = 0$

$\therefore 84.375\,x - 24\dfrac{x^3}{3} = 0 \Rightarrow x = 3.25\,m$

D = 3.25 + 0.875 = 4.12 m

Penetration depth = 1.2D = 4.95 m



Point of max. moment (shear=0)

$\sum F_x = 0$

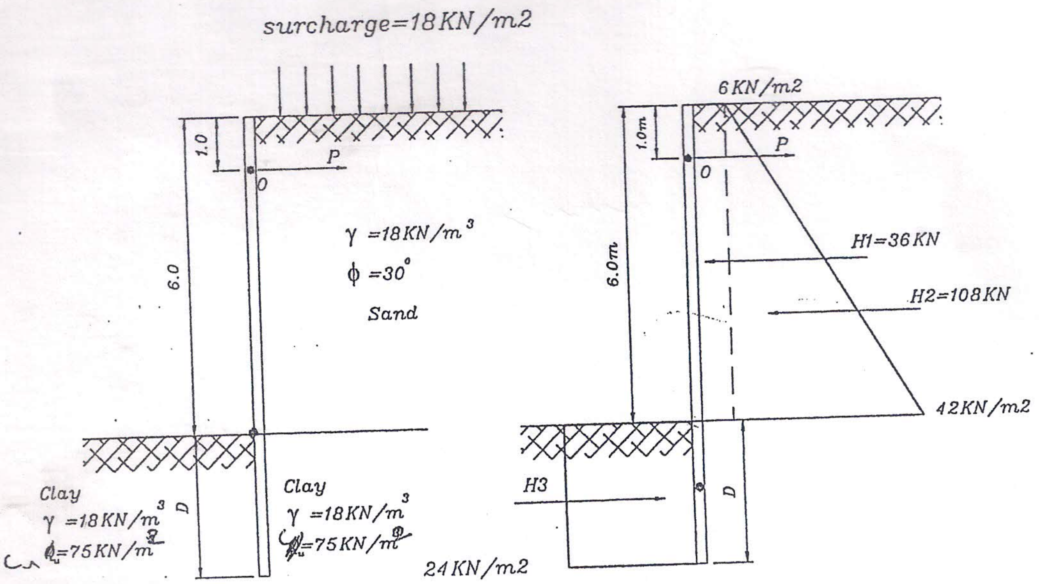
$\therefore 6Z + \dfrac{1}{2} \times 6Z \times Z - 78.42 = 0$

$3Z^2 + 6Z - 78.42 = 0 \Rightarrow Z = 4.21m$

$M_{max} = 78.42 \times (4.21 - 1) - 6 \times \dfrac{4.21^2}{2} - \dfrac{1}{2} \times 6 \times 4.21 \times \dfrac{4.21^2}{3} = 123.94\, kN.m$

Problem # (27)

Refer to problem 25, what would be the depth of penetration, maximum bending moment and anchored pull if the soil below dredge line was a clay deposit with a unit weight of 18 kN/m^3 and unconfined compressive strength of 75 kN/m^2 **(Use free earth support method)**



Solution:

Force (KN)	Arm about anchor, point O (m)	Moment (kN.m)
H_1= 36	2	72
H_2= 108	3	324
H_3= -24D	5+0.5 D	-120D –12D^2
		-12D^2-120D+396

Taking the moment about the anchor equal zero

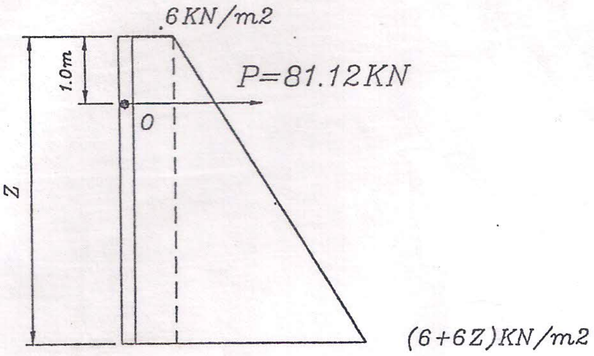
$$\sum M_O = -12D^2 - 120\,D + 396 = 0 \Rightarrow D = 2.62\,m$$

Penetration depth = 2.62 m

$$\sum F_x = 0$$

$$\therefore 36 + 108 - 24 \times 2.62 - P = 0$$

$\therefore P = 81.12 kN$ **per meter width of the wall**



Point of max. moment
(shear=0)

$$\sum F_x = 0$$

$$\therefore 6Z + \frac{1}{2} \times 6Z \times Z - 81.12 = 0$$

$$3Z^2 + 6Z - 81.12 = 0 \Rightarrow Z = 4.29\,\text{m}$$

$$M_{max} = 81.12 \times (4.29 - 1) - 6 \times \frac{4.29^2}{2} - \frac{1}{2} \times 6 \times 4.29 \times \frac{4.29^2}{3} = 132.72\,kN.m$$